The recreaMATHS **Methodological Guide**

recreaMATHS

Contents

Chapter 1 4

An introduction to the concept of non-formal Mathematics 4

1.1 Non-Formal Mathematics 4

1.2 Mathematics in Kindergarten 6

Chapter 2 16

European Museums of non-formal Mathematics 16

2.1 4 European Museums of non-formal Mathematics 16

2.2 A Museum's Aspiration 23

2.3 Conclusions deriving from the project partners' discussion with math museums' experts 25

2.4 Understanding Mathematics as a socio-cultural product; the linkage of Mathematics with the European Cultural Heritage 33

Chapter 3 38

Adoption of digitized methodologies that appeal to kindergarten educators to broaden and enrich math experiences in kindergarten classrooms 38

3.1 Digitized methodologies to teach Mathematics in Kindergarten 42

3.2 Employing digitized tools and technological innovation for the creation of educational material by the kindergarten educators with the aim to reinforce mathematical learning in kindergartens 47

3.3 Using technological innovation for the training of kindergarten educators 58

Chapter 4 63

Alternative pedagogical methodologies and synchronous interdisciplinary best-practices to approaching simple math concepts and reasoning for preschool ages 63

4.1 What is interdisciplinary learning and why is it important in preschool? 63

4.2 Approaching Mathematics in Kindergarten as a multifaceted and interdisciplinary field 66

4.3 Inclusive Math Approaches 72

4.4 Alternative pedagogical approaches 81

4.5 Best Practices to approach simple math concepts 86

Chapter 5 89

The recreaMATHS approach 89

5.1 Understanding the recreaMATHS approach 89

5.2 Problem-Solving Skills as a recreaMATHS approach 101

5.3 Teach children to view and describe their world mathematically 111

References: 116

Chapter 1

An introduction to the concept of non-formal Mathematics

1.1 Non-Formal Mathematics

How can we define the concept of non-formal learning/education?

The term 'non-formal education' goes way back, to 1974, where Coombs and Ahmed used the term for the first time. Coombs and Ahmed, determined that learning and education could be equated, despite how, where, and when the learning process occurred (Mok, 2011). **Non-formal learning can be defined as a form of learning which occurs outside the classroom, separate from the formal school system**. In other words, outside the parameters of traditional learning institutions and structures. Thus, an educator together with a student, 'hold' their activities and learning outside the formal system. The terms community learning, adult education, lifelong learning can be used interchangeably in non-formal education (Khasnabis et al., 2010). Non-formal learning is about, recognizing how important learning, education, and training outside the standard educational foundations, are (*What Is Non-Formal Education?*, 2015). Moreover, non-formal education is used in the procedure of lifelong education of individuals, as an addition, an alternative approach, or as a complementary learning method to formal education. It is not used as a replacement for formal approaches. Non-formal learning guarantees providing access to education to all individuals of all ages (*Non-Formal Education*, 2020). Thus, non-formal education and learning refer to a relatively methodical type of learning, which is not necessarily pre-planned. Similarly to formal learning, non-formal has as a target for learners and teachers to achieve specific learning tasks (Mok, 2011).

As we all know, there are different methods when teaching different learners, and different topics that can be taught in a different setting with a different approach. By introducing non-formal learning, teachers and students become equal. In other words, there is no need for students to call the teacher 'Sir' or 'Miss' and a student's schedule is as important as the teacher's schedule. Non-formal learning focuses on the empowerment of a learner in achieving more and challenges both the student and the educator to come up with a variety of ideas, to listen as well as argue with each other (Spiteri, 2016). It is important for all of us, to recognize non-formal learning and education as an indispensable part of the process of education and acknowledge the influence non-formal learning can make in educational organizations. Non-formal learning can be considered as an essential part of the concept of lifelong learning and can ensure that learners preserve the skills and abilities which are needed in adapting to a repeatedly changing environment. The collection of non-formal teaching tools and different learning structures can be seen as an innovative and creative alternative to the classic and traditional teaching schemes. The benefits of introducing non-formal approaches include getting a chance to experiment and take responsibilities through engaging in non-formal education; being able to develop enthusiasm and curiosity towards the learning process; learning to work together in a team, and build decision-making skills. Furthermore, an educational process based on non-formal learning can help in the development of personal and social skills, through the experimental process (*Non-Formal Education*, 1999). Through the development of the personal and social skills of each individual, teachers can help the children boost their self-esteem. A healthy critical attitude of the surroundings can be developed by each individual's learning and 'discovering' capacity ("What Is Non-Formal Education and Why It Is Important," 2018).

1.2 Mathematics in Kindergarten

Kindergarten math has as a main goal to prepare preschool children for the mathematics they will come across in first grade. Preparing children for this next step requires much more than just handing children worksheets and books. Young children will start to recognize and understand abstract concepts and symbols after they experience ideas. These experiences will integrate their senses by experimenting and making observations which allows them to examine a topic even further. Children learn math by grasping the concepts at their own pace. It is suggested that children return to previous tasks and try to solve them using a distinct way. Moreover, in order for preschool children to understand new math concepts and abstract ideas, they need to practice using concrete objects, such as blocks, sticks, counters, etc. Teachers should ensure that before being able to use the mathematical methods for guided math activities, children take a sufficient amount of time. Using mathematically based activities and games is a good opportunity for children to build a math vocabulary as well as connect mathematics to their everyday experiences ("How to Teach Kindergarten and Preschool Math," 2019).

The **playful approach to math** – Modern approaches in teaching mathematics:

The concept of play is usually considered as a less academic activity and is frequently limited to young children and students when learning mathematics. On the other side, mathematics is known as a disciplined, logical, and boring subject – as considered by students most of the time. However, since learning through play is an acceptable pedagogical approach in kindergarten, educators should set up their teaching on this learning approach. While playing, children may reach a state called "flow" – an indefinable state of mind where time seems like disappearing when being deeply focused on what they are doing. To get to this optimal state, children's mind requires freedom of play but also, a reaction from teachers to children's ideas, and guidance through notions such as numbers and counting. Offering direction while creating freedom allows productive play which helps children open their minds and understand better the more difficult mathematical concepts. We can conclude that the amount of play in mathematics, a "serious" abstract subject, is inversely proportional to the age of the children/students. This means that, the older the children the less the amount of play in the process of learning mathematics. Nonetheless, this does not have to be the case (Oldridge, 2019).

We aim to create a culture where ideas based on mathematics are not 'just formulae on a page' but instead, discussed and reasoned through concepts. This encourages children to reason, talk, think, and wonder as they go through a problem while creating a sense of curiosity – even for simple and easy concepts – which helps to playfully engage children. For most problems, there is no single approach or path to its solution, and this generates an interesting, surprising, and fun side of mathematics. Thus, teachers should be open to different thinking and to new ways used in solving a problem, to therefore adopt a culture of curiosity in openness to the unforeseen. In order for children to set a mindset that helps them improve their comprehension of complex math concepts, teachers could set

a playful, hopeful, and optimistic math approach. Observing the children as they work on their own or in teams helps the teacher understand how the children are adapting to new challenges. Furthermore, to improve math teaching, listen, and talk to your children while they play/work. This will allow you to understand each child's way of thinking in the problem-solving process. Play generates spaces, which are open for thinking where children can be guided in engaging with interesting and exciting concepts of mathematics (Oldridge, 2019).

Improving the teaching of mathematics in kindergarten:

Teachers may use as a starting point for their preparation and decision making, their students' capabilities. Resources that can be used as a starting point include mathematical reasoning, language, listening and reading skills, as well as being able to cope with complex concepts. For children to develop context-related strategies, they need to envision the situation/events where the problem is set in. In this case, they can use their knowledge and experiences as the key basis for the aforementioned development of strategies. Instead of rejecting an 'alternative interpretation of mathematical ideas', and categorizing it as "wrong thinking", teachers can alternatively view them as an ordinary and necessary step in the learner's development of concept formation. Teachers can provide several opportunities for them to learn from their errors. For instance, by formulating a discussion where children's attention focuses on the difficulties that have appeared or by asking the children to share their thinking, their understandings, or solution strategies to compare and re-examine their solution. Kindergarten children develop ideas about mathematical concepts by engaging with math-related tasks, which helps them discover the scope of making sense of mathematics. Furthermore, learning experiences allowing original thinking, encourage students to be capable learners of important mathematical concepts. Instead of encouraging single-minded tasks, teachers should offer opportunities to

children to struggle with concepts so that a variety of increasingly sophisticated mathematical processes is developed. Children should be encouraged and supported by teachers in creating connections between the distinct ways of problem-solving, between mathematical concepts and representations, and between mathematical concepts and everyday experiences (Anthony & Walshaw, n.d.).

It is necessary for educators to understand how the learning of mathematics is endorsed by young children's engagement in play, as well as how children can support best this learning. For example, a child's learning can be maximized if adults help them reflect and represent on their everyday experiences. Moreover, 'learning through play' is considered as a fundamentally good pedagogy in the learning of mathematics for young children. A "good" mathematics pedagogy includes a math-talk promotion, a productive disposition development, mathematical modeling emphasis, the use of tasks that are cognitively challenging, and a formative assessment. A good mathematics pedagogy can be endorsed when teachers engage and encourage children in activities across distinct areas of learning through a variety of mathematically-related activities. The activities should be child-initiated; in other words, rise from a child's interest, concerns, questions, and everyday experiences. The features describing a good pedagogy require a deep understanding and should advise the distinct ways on how educators can engage kindergarten children in such mathematically-related activities such as play, project work, story reading, physical, and art education. When teachers focus on a child's mathematical sense-making, can best realize the potential of such engaging activities for the development of mathematical proficiency. Furthermore, one of recreaMATHS aims is to maximize opportunities that engage children by providing a range of tools, such as digital tools to help in facilitating learning (Dooley et al., n.d.).

The foundation of a child's very first mathematical experiences is a child's play and its interests. Most children's preschool mathematics learning takes place during play or playful activities that deliver the key contexts of mathematics. During the children's free play, they may spontaneously engage in several mathematical concepts, which some of them can be quite advanced at some levels. Children may even play with math itself. The idea of the play depicts a context where children are able to reflect on their former experiences, connect experiences together, represent the experiences in distinct ways, explore different possibilities and create meaning out of them. Mathematical thinking has a strong connection with these procedures and may be inspired by children's experiences. Mathematical language and notions are encouraged through the context of the play. Through this context of play, children can discover discrete mathematical ideas while teachers can be provided with a context that develops and supports the children's ideas. Teachers – adults in general – have a critical role with the children they interact with. Their role is to help children reflect, as well as help them talk about their experiences while playing, to help them maximize their learning potential. This promotes and encourages children to think mathematically including mathematical learning. From this point of view, sensitive structuring of a child's play as well as learning through play can be both seen as an important element to a good mathematical pedagogy of young children (Dooley et al., n.d.).

Source: https://mcosnon.wordpress.com/2016/02/02/logical-mathematical-intelligence-numberreasoning-smart/

Key points summarised:

- For educators to comprehend the way the learning of mathematics is promoted through the engagement of young children while playing, as well as how teachers can best support this specific learning.
- Good mathematical pedagogy features can be acknowledged with reference to strong principles which relate people and relationships, and a learner's environment with the learner.
- The features along with the principles of a good mathematical pedagogy for kindergarten children (aged 4-7 years) relate to a variety of early educational settings which, on their side, are significant in the promotion of continuity in the pedagogical methods across all distinct settings (Dooley et al., n.d.).

An example: The **perception of mathematics** in a Swedish preschool:

In 1998 the first curriculum-based iteration was produced, and since then the Swedish preschool curriculum has had clearly specified objectives about mathematics. Nonetheless, the aims are not objectives to be reached by children, but rather to be strived by the preschool itself. The emphasis of mathematical concepts was for teachers and preschool staff in general, to notice and consider mathematics in the everyday circumstances of preschool. In other words, to better comprehend their own views about mathematical concepts and to assess the contribution offered by the preschool on children's mathematical development (Johansson, 2015).

Examples of mathematical concepts taught in a Swedish preschool – a curriculum change:

The main aims of the preschool are to ensure that each of the children:

- Develops an understanding of concepts such as shapes, space, direction, and location, as well as the basic properties of quantity, order, sets, and number and lastly notions such as direction, change, and time;

- Develops the ability to investigate using mathematical concepts and reflect over, as well as test, the different solutions of problems which were raised by themselves or by other children;
- Develops the ability of expressing, examining, distinguishing, and using mathematical concepts and their interrelationships;
- Develops math-related skills 'in putting forward and the following reasoning'.

In order to develop further the debate of 'what mathematics for young children in preschool should be', Johansson continues with linking social practice to cultural practice. The author discusses that mathematical activity and mathematical practice are both cultural. Thus, the mathematics that takes form in this culture can be derived by dealing with quantities and the spatial understanding of the environment. A cultural understanding of mathematical concepts can be connected to the children's perspective in developing a mathematical understanding (Johansson, 2015).

Mathematics in a **cultural context**:

We can say that the learning of mathematics, explicitly, is formed by the shared understandings of one's culture. In other words, for a child to comprehend mathematics and know how to express the knowledge gained in the classroom, needs to explore all distinct paths. Moreover, language and culture can play a tremendous role in the way a child learns to count (*Making the Connection between Culture and Mathematics Northwestern University | School of Education & Social Policy*, n.d.).

What are **Art and Culture** and how can it be linked with mathematics?

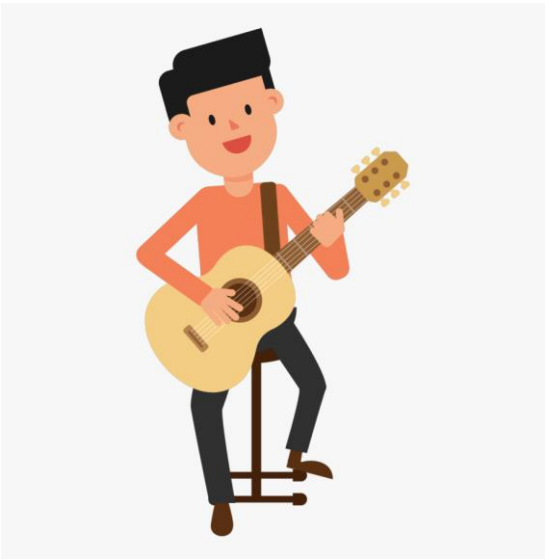
Culture can be defined as a set of ideas, customs, and social behavior as well as the characteristics and knowledge of a specific group of people (*What Is Culture? Definition, Meaning and Examples* | *Live Science*, n.d.). Most of the educators teaching subjects such as mathematics, assume that mathematics is a non-cultural subject. However, mathematics is not a culture-free pedagogy. In some way, mathematics can be considered as an essential component of all cultural contexts (d'Entremont, 2015). In this case we will use as an example of culture, art – defined as a huge subdivision of culture, broken down into several creative activities and disciplines ("Culture and the arts", 2019). Dooley et al, examine the learning of mathematics through the arts, which as mentioned above, is a vast subdivision of culture. They discuss specific ways in which links between mathematics and the arts (visual arts, music, and drama) can be established. Some examples include the following. Teachers can use the rich context of culture in music to develop a child's mathematical concepts and language. Through the classification of sounds and movement, children can enhance their mathematical skills and understanding. Moreover, a strong link between timing, order, rhythm, and beat of the music, and the concepts of mathematics such as sequencing, counting, can be identified. By engaging children to music, helps them in the development of other attitudes and skills important to mathematics. This includes concentration, perseverance, creativity, sensitivity and self-confidence towards other individuals.

Moving on to visual arts and mathematics. Both shapes and patterns are important features of mathematics and visual arts. A significant aim of the visual arts curriculum is the development of a child's awareness of, enjoyment to and sensitivity of visual, tactile, aural and spatial environments, while also important is the awareness of the spatial and visual qualities in the environment for the mathematical understanding. Likewise, the enhancement of a child's ability to apply mathematical knowledge in real

life and in the environment is as significant (Dooley et al., n.d.). For instance, in France, there is an increasing tendency to have distinct disciplines interact on the same subject [e.g., teaching Math, Physics, and Technology while creating/building a weather station].

Mathematical notions based on cultural perspective as well as art allow children to appreciate and reflect on their own culture while also, appreciating the traditions and culture of others. To benefit from these rich cultural experiences implies that students are exposed to several experiences as well as cultural resources. Kindergartens could set as an aim, to help children learn about their culture and the culture of others through activities which determine the connection between mathematics and culture (d'Entremont, 2015). To expand a child's, an educator's and the parents' views about preschool math, is not a straightforward procedure and should take time to accomplish this process (Johansson, 2015).

"Mathematics is a way of thinking and understanding our lives and our world. It is a set of tools, a pair of glasses that we can use"*, Hyde & Bizar(1989).*



Source: https://oakfieldschoolsfederation.org/2020/05/read-to-sing-part-2/

There is a continual increase in the kindergarten mathematical education as well as an increase in the early childhood education in general.

Several research findings confirm that teaching mathematical concepts in kindergarten education can facilitate the transition from non-formal kindergarten mathematics to formal elementary mathematics by providing cognitive foundations in children's capability to become skilled in the systematic teaching of "real" mathematical concepts in advanced educational stages. The low performance of children internationally in the subject of mathematics, reflects the need for a distinct method to teaching the mathematical notions, being different from the traditional method to learning and teaching mathematics. Kindergarten children come to primary school with knowledge based on informal numeracy which can be extended, developed and enhanced through appropriately designed learning activities. Subsequently, kindergarten educators can make the teaching of mathematics more interesting by emphasizing in creating an innovative and different learning environment (Papadakis et al., 2016).

Chapter 2

European Museums of non-formal Mathematics

2.1 4 European Museums of non-formal Mathematics

- In Germany **MathematiKum**

Mathematikum is a mathematics museum, located in Gießen, in Germany. It was founded in 2002 on the initiative of Albrecht Beutelspacher who, organising a seminar for future teachers, gave them the task of creating a geometric model and explaining its mathematical functioning. Their commitment and enthusiasm made the professor of geometry realise the importance of founding an innovative museum in the fields of mathematics. After several temporary exhibitions, Abrecht Beutelspacher was awarded a major prize by the Stifterverband für die Deutsche Wissenschaft, an association that seeks to identify and meet challenges in higher education, science and research. The idea of a "German mathematics museum" was finally recognised by the nation and the project was launched in 2002.

As soon as it opened, the attendance figures exceeded all their expectations, they expected 60,000 visitors, and more than double that number visited the museum in the first year. In 2010, the million-visitor mark was even passed, showing that public interest has not changed over time.

The Mathematikum museum is based on three pillars: mathematics for all (all ages, all categories, with or without a passion for mathematics); cooperative mathematics (where people can help each other); and, finally, practical and fun mathematics (where people experiment and manipulate).

This museum offers multiple interactive experiments to better understand mathematical concepts (from geometry to functions and probabilities). These experiments are designed to be experienced by all ages and to stimulate all of our senses: we are offered to lock ourselves in a giant soap bubble, to measure our size with the binary system, or to see ourselves an infinite number of times. More than 170 new experiences are spread over $1200m^2$ of floor space.

The success of the museum is explained by the fact that visitors are really taken into account and are a significant part of their visits: they can move freely and leave at any time. Furthermore, the mathematics presented are taken seriously and the material conditions (buildings and facilities) are subject to extreme vigilance (they are regularly maintained and restored).

At the same time, the Mathematikum is developing its traveling exhibitions in Germany, France, and even more widely in Europe. There are three of them: the first, "Hands-on mathematics", presents a selection of the most popular experiments; the second, "Mini-Mathematikum", follows the general concept of the Mathematikum and is accessible from the age of 3; and finally, the last, "What a coincidence! "offers experiments on the theme of coincidence.

Source: Mathematikum

- In Spain, **MMACA**

The "Museu de Matemàtiques de Catalunya" (MMACA) was created in 2006 in Cornellà de Llobregat, Catalonia. This museum is the only mathematics museum in Spain and therefore represents a great opportunity to unite and bring together its citizens around mathematics, bringing them together through interactive experiences and manipulative activities.

When it was created in 2006, the museum only had travelling exhibitions, i.e., it did not receive the public onsite but made its activities travel around Spain. Today, there are more than ten travelling exhibitions, with different levels of difficulty and on different themes such as intuition, geometry or the tangram.

Alongside this activity, MMACA has opened its doors on the upper floor of the Mercader de Palau Museum in Cornellà since 2014. It covers an area of more than $300m^2$, where you can find its permanent exhibition "Mathematical Experiments". This exhibition is made up of games, construction games and a series of elements related to geometry, calculations and statistics or strategy. One of the museum's mottos is: "*Forbidden NOT to touch*", because the important thing for this museum is that people can manipulate and experiment with the elements they are offered and perhaps change their minds about mathematics.

With the aim of reaching out to all audiences, the Mathematics Museum of Catalonia seeks to cover both the playful and the rigorous aspects of mathematics so that the Catalan mathematics scene continues to support it.

In addition, MMACA is increasing their European and international projects in order to support the European mathematical culture and to participate in the development of frameworks that will subsequently favour the learning and understanding of mathematics by students.

Source: Museu de Matematiques de Catalunya

- In Sweden, **Navet Science Centre**

The Navet Science Centre is one of four municipal associations in Borås Sjuhärad. This "informal region" was established in 1999 when a new county - Västra Götaland - was created with the aim of safeguarding the interests of the municipalities in the Borås sub-region. The region's aim in establishing the four municipal associations was to create a spirit of

initiative, ingenuity, creativity and entrepreneurship among its 280,000 inhabitants. The Navet Science Centre was built precisely to participate in and support the development of the skills of the school staff by designing teaching methods and materials.

This science centre covers more than 3500m^2 and raises awareness on a wide range of subjects such as mathematics, astronomy, creative technology, space creation and the environment (from climate to food and agriculture). It seeks to reach out to all age groups by expanding its offer and allowing anyone to manipulate, in order to offer continuous training of the different subjects. For instance, offer training on the environment for companies seeking to learn more about the subject, but also offer teachers training which can be further explained to their students. Students of all ages (from kindergarten to high school) can also come to the centre to do activities involving mathematics and other scientific fields. In addition to its permanent exhibition, the Navet Science Centre also offers several travelling exhibitions for rental, particularly on the subjects of geometry, algebra, and probability. This allows the knowledge acquired during the permanent visit to be deepened and internalized. Teachers particularly appreciate this offer and many have now fully integrated it into their teaching: it allows them to discover a new way of presenting science and to explain it more clearly to their pupils.

The Science Centre is one of the 19 Swedish science centres grouped together by the association Svenska Science Centres with the aim of stimulating scientific curiosity. According to the 2011 Science Centre World Congress statement, these centres and their scientific commitments are based on three pillars: scientific knowledge, hands-on interaction and co-creation of experiments with scientists and the public. Each year, the National Agency for Education checks the quality of the science centres' offer before offering them a national grant or not. In 2016, 2017, 2018 and 2019,

the Navet Science Centre scored the first place, with a total of 35 points out of a possible 40 (in 2019).

Source: Navet Science Center

- In France, **Fermat Science**

Fermat Science was founded in 1996 in the town of Beaumont-de-Lomagne, near Toulouse, following the excitement and enthusiasm due to Andrew Wiles' demonstration of "Fermat's great theorem" in 1995. From its inception, Fermat Science has sought to promote mathematical culture in a fun way to all audiences while linking this field to other cultural or heritage knowledge. Indeed, the association relies on the history of its town and surroundings as well as on the history of Pierre de Fermat to enrich and diversify its offer.

Fermat Science offers numerous workshops in the Maison de Fermat, the place where Pierre de Fermat was born and lived occasionally. These workshops always involve one or more mathematical concepts and can concern both the public school (from kindergarten to high school) and the general public. To name a few, the workshops may focus on maths and

art like optical illusions, origami, stained glass and mosaics, or on maths and cryptology or maths and puzzles, such as tangrams, pavements, labyrinths, etc. Fermat Science also travels and offers these same workshops in schools. Some of these workshops are also available for rental and travel to schools, colleges and high schools all around France or other foreign countries. For instance, the exhibitions *Voyage en Mathematique* or *Curiosités Mathématiques*, or kits like *Divertimaths, Festimaths* etc. In addition, the association presents an annual exhibition on a theme related to mathematics such as coincidence, geometry or weather and climate.

The association organises events such as *La fête des Maths*, *Femmes en Sciences 82*, *Matermaths* or *bâtiTMaths*. These events aim to give a different image of mathematics on specific themes for different audiences. Moreover, Fermat Science, anchored in the local heritage of its surroundings, also seeks to make its contribution to the dissemination of mathematical culture on a European scale. To this end, the association participates in European projects such as STEAMbuilders and Math Reality.

Source: Fermat Science

2.2 A Museum's Aspiration

Places of scientific heritage and culture are constantly redefining themselves and rethinking their role in and for society as well as in their relationship to the public. If the notion of "inclusive museum" originated in the early 1980s in an Anglo-Saxon professional community, this posture is now widespread in the French-speaking and European space. Beyond the accessibility of certain spaces or the adaptation of systems, the museum space is open to everyone, inside and outside the walls, in urban, peri-urban or rural areas.

Museums of all categories are constantly redefining themselves socially. The notion of inclusion is particularly questioned by museums at the level of their institutional and public policy, but also of their cultural and scientific communication project, their managerial policy and their economic model. Over the last twenty years or so, there have been numerous initiatives to include and make room for different audiences in museums, thus creating new avenues for reflection on professional practices and the evolution of professions. As an actor for and in society, the museum is, for example, developing participative mediation, working on accessibility, questioning the representations of professionals and the public to be "included". Partnerships with cultural, health and economic structures and associations are being created.

Access to "culture for everyone" is one of the objectives of democratisation. Making the place accessible and open to all people regardless of their social origin, level of knowledge, or health becomes an objective, in the name of the general interest. It concerns all cultural and museum structures that are part of the public or associative service. Individuals and citizens are equal and as such share common interests.

The care taken in welcoming the public is reflected as much in the design process as in the first moments of the meeting. Indeed, from the very beginning, it is a question of creating confidence in order to put visitors at ease. This consideration of the public continues afterwards by adapting the discourse and a capacity to listen and be open.

The social and inclusive museum is a place where sharing and exchange are privileged between the institution and the individual whether they are local, disabled, socially distant or a tourist. A place for reflection and debate, in search of dialogue, highlights the identity of territories, their rich heritage, cultures, people and their knowledge.

For students, the museum can help to motivate the study of mathematics, from the point of view of the emotional approach. But it can also provide insights and tools to enrich those experiences that are the basis of the formation of the mathematical idea.

If we start from the consideration that a session in the museum must be special, exciting and motivating and essentially different from a school activity, it is clear that the exhibits of an exhibition must be "special" by size, design, materials and interaction.

In this case, the main goal must be to keep in touch and/or inspire class activities and contents.

- Materials must be simple, easy to create.
- If possible, easy to be built by students themselves.
- New technologies, as 3D printers or the FabLabs network, can deeply and quickly increase the possibilities to self-produce pull-up exhibitions.

However, an exhibit must immediately motivate an action (hands-on or virtual) and communicate the basic concept that inspires it, requiring the minimum instruction and intervention by the educator.

On the contrary, an exhibit has to stir discussion, exchange and collaboration among the users, to develop into the basic elements of making a visit to the exhibition a real moment of learning.

Additional information (historical placement, uses, concepts involved ...) can be present as elements of the exhibition (panels, short videos, still or moving images, simulations on a computer ...) or entrusted to a virtual visit through the museum's web page or a follow-up period at school or at home, either individually or in groups.

Finally, a reflection is needed about the concept of proximity: with materials, activities, relationship with the public and the territory, but also, if not above all, with the concepts and the methods of mathematics. Most of the society considers maths abstract, dry and distant. It is then fundamental that, without trivializing or spectacularising it, we can highlight aspects that bring mathematics to each one's experience and stimulate the pleasurable aspects of discovery, resolution, and improvement of self-performance.

2.3 Conclusions deriving from the project partners' discussion with math museums' experts

This report is intended to provide a contributive, comprehensive, and thorough preview of the scope, collection, and educational opportunities of Math Museums. The consortium of the project has connected with the 4 prementioned Math Museums from 4 distinct European countries – Fermat Science in France, MMACA in Spain, Mathematikum in Germany, and Navet in Sweden – to contribute and to render assistance to the provision of necessary information for the recreaMATHS project. The Math Museums' experts were provided with a questionnaire of 5 sections: Museum Details, Museum Scope-Aims-Visions, The museum's collection and exhibits, Museum Education and Training, and Mathematics at the museum. Despite all of

them being museums with an identical area of expertise – being mathematics – the responses of all four museums are inspirational and significant. The creation of a report based on Math Museums aims to increase educators' understanding of the museums' aspirations and to popularize the educational methodologies and pedagogies developed in a museum.

The following results were obtained by a google form questionnaire:

Museum Name	Country	Year of establishment	Area of Expertise	How many (on average) preschool children visit the museum
Fermat Science	France	1995	Mathematics	5400 students (2019)
MMACA	Spain	2014	Mathematics / Hands-on material	10000
Mathematikum	Germany	2002	Mathematics	15000
Navet Science Center	Sweden	1996	Mathematics and Science	2000

How about we have a broader perspective of the museums' responses and focus individually on each question, starting with section 2 and the museum's scope. Fermat Science's scope is to introduce school children to a non-formal approach for learning mathematics through the use of several mathematical activities, such as games, visits, exhibitions, and workshops. The museum has an aim to create an interactive mathematical space in the birthplace of Pierre Fermat, to popularize mathematics with non-formal tools, to promote and disseminate the topic of mathematics and create

educational tools. Similarly, MMACA in Spain has a scope to promote the topic of mathematics and offer visitors a deep and exceptional mathematical experience. They aim to promote mathematics, show the kind and powerful aspect of mathematics, offer a joyful math experience as well as provide a good vision of mathematics and support for their teachers. Moving on to Germany, the Mathematikum museum has set its scope to give the opportunity to as many people as possible – especially younger people – to participate in mathematics. Last, Navet Science Center's scope is a 3500 square meter exhibition space, focusing on topics such as mathematics, water, astronomy, creative technology, maker space, sustainability Hub – among other things; digital hybrid arena, 3 labs, facilities for in-service teacher training. Navet's aims include inspiring the curiosity and the will of all visitors to learn, being an inspiration for mathematics, science, and technology, helping teachers and schools develop new methods for teaching in mathematics, science and technology, and encouraging entrepreneurial learning and action.

Mathematical Experience

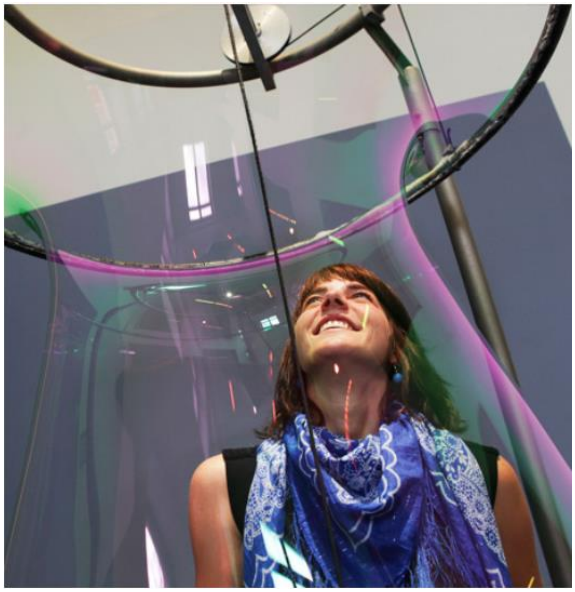
Source: *MMACA museum*

Besides the museums' aims, the visions that each museum has is as important. Fermat Science museum's vision includes, the popularization of mathematics, to show their objects and how they work in both historical and recent discoveries, presenting mathematics in a fun way that is accessible to as many people as possible, and emphasizing the links of mathematics with

other fields of knowledge and culture. The visions of MMACA and Mathematikum respectively, are to develop a pleasant reputation and material which are highly rated, and for people (families, friends, etc.) to come in their leisure time to "do math" and leave the museum happier. Finally, Navet Science's vision is to be a noticeable component and collaboration partner for the realization of the sustainable development goals.

Proceeding to section 3, the museums were asked about their main collection of exhibits. Fermat Science responded that at the moment they have the Szilassi and Pierre Fermat sculptures, exhibits of Fermat called 'enfant de la Lomagne', 'Voyage en mathématique' and 'Curiosités Mathématiques'. The most acknowledged exhibits of the museum for kindergarten children are mostly based on experiencing hands-on because at such an age touching and handling are very important. Exhibits include a space for games and manipulations, a mathematical manipulation tool in the form of challenges, a mathematical workshop with big geometric shapes. MMACA's main collection of exhibits consists of some of Pythagoras' visual proofs, joyful puzzles and enigmas, polyhedral mirrors, dissection of geometric shapes, and optical illusions. Kindergarten children's favorite exhibit is to build free structures or polyhedrons using big geometrical pieces. Moving on to the main collection of exhibits of Mathematikum. The museum's exhibits range from puzzles – such as Conway's cube – over the investigation of an object y – for instance light and shadow or mirrors – to 'big' exhibits, such as a turntable, giant soap film, as well as very few computer-based exhibits with a nice and simple surface. Children have different favorite exhibits, with some of them playing "building a city" and some playing the gear-wheels or even lying in the mirror house. Lastly, Navet Science Center's main collection of exhibits includes Berta the Dragon's chemistry lab, the sustainability Hub, Bagdad and the infinity,

Crime lab, Maker space, and Astronoma (an astronomy exhibition). The most acknowledged exhibit of the museum is Berta the Dragon's chemistry lab because there are not many possibilities for kindergarten children to experience real chemistry.



The Giant Soap Film

Source: *Mathematikum museum*

Further, we should also consider how the museums are connected to European cultural heritage. Fermat Science is linked with the European cultural heritage through the internationally recognized famous mathematician Pierre Fermat who communicated and worked with European scientists of his century. Furthermore, Mathematikum is linked to European cultural heritage by preserving and disseminating the incredible cultural heritage of mathematics of the last 5000 years, particularly Greek – but not exclusively – Mesopotamian, Chinese, Mayan, all of whom have developed breathtaking mathematics. Navet museum works with the following components of European cultural heritage: tourism and heritage in promoting sustainable cultural tourism, engaging the younger generation, heritage-related skills in enhancing education and training for the traditional and new professions, fostering participation and social innovation as well as using research, innovation, science and technology for the better conservation and presentation of heritage.

Moving on, the partners were asked to give 2 examples of exhibits that can be easily modelled and printed by kindergarten educators, and additionally if the museum's collection supports inclusive education. In other words, to recognize all children's entitlement to a learning experience that respects diversity enables participation, removes barriers and considers a variety of learning needs and preferences. Starting with Fermat Science, the museum suggested two exhibits that could be 3D printed, La chasse aux forms (The hunt for shapes) and Les apprentis géomètres (Apprentice Surveyors) which is an educational tool created with the help of national education specialists for inclusive education. MMACA museum suggested an exhibit for matching shapes and colors, Tangram and "Patrons". All of the museum's exhibits are designed for inclusive education; the exhibits are free of barriers and some exhibits are even suitable for blind people. Similarly, Mathematikum in Germany embraces a collection of exhibits supporting inclusive education as well as Navet in Sweden. The Swedish museum is constantly working with the problems of inclusion and lately, they have even worked with groups of mentally disabled individuals through the use of creative technology, immigrant families in topics such as sustainability and art, and the museum is working long term with groups of children who would have limited access to Navet museum thru their families.

Hence, the museums were asked to describe the educational opportunities they offer, and if they have a rich array of training offers for kindergartens. According to Fermat Science, the museum offers school activities such as workshops and games where students manipulate several mathematical concepts learned in class. Fermat offers the following training workshops for kindergartens: shapes, colors, numbers, algorithms and labyrinths, spatial geometry, shape hunting. Furthermore, they propose a game of recognizing shapes in Beaumont de Lomagne city as well as construction games. MMACA in Spain offers an opportunity to experience a

new and different understanding in schools. The museum has some didactic suitcases with hands-on material to bring to school and can use them for pop-up exhibitions or even for creating some activities for the classroom.

Tangram for kids

Source: *Fermat Science*

Moving on to Mathematikum; several times a year the museum invites kindergarten educators for a course and offers approximately 30 courses that connect "Haus der kleinen Forscher" (House of Little Researchers) to STEM topics. Finally, Navet Science Center offers in-service and pre-service training for teachers, public events in local communities, kindergartens and schools, students of all ages, outdoor facilities and activities for all, city events, sustainability training for companies and other organizations, etc. The museum offers many programs for kindergarten teachers, for instance, programming and digital tools, chemistry, physics, mathematics and so on. Navet considers that working with younger preschool children on mathematics is a better way to pursue such concepts "in-house".

The museums cover a number of preschool mathematical concepts. Concepts such as shapes and numbers are covered by all four museums, yet through different exhibitions and workshops. The following table portrays the distinct concepts covered by each Math Museum:

Museum	Mathematical Concepts covered in museum's exhibits
Fermat Science	geometry shapes, counting, colors, and find one's way in space
MMACA	shapes and forms, number concepts, early operations, reasoning, solving problems, and creativity
Mathematikum	numbers, shapes, functions (movement), as well as metacontexts such as communicating, arguing, describing, etc.
Navet Science Center	games, puzzles, measurements, digital math and counting

The last section of the questionnaire is about the mathematics at the museum, how effective is the teaching of non-formal mathematics to preschool children and if the children leave with a different perspective towards mathematics after visiting the museum. As stated by Fermat, when teaching non-formal mathematics, it is best to manipulate appropriate mathematical concepts since this approach always succeeds. According to MMACA, non-formal mathematics offers a really good and pleasant mathematical experience which is difficult to achieve at school and this is, therefore, very important for the future development of mathematical skills. Mathematikum believes that the teaching of non-formal mathematics to kindergarten children is a very effective method and is definitely much more powerful than teaching mathematical syntax. Nonetheless, Navet museum thinks that non-formal mathematics is more effective when carried out on a children's every-day basis at kindergarten or in outdoor facilities. In the long run, the best outcome comes from training the teachers that work in kindergartens and support the children's work. After visiting the museum,

children have a different view towards mathematics, and this can be confirmed by all museums. The children have experienced a new view of the world, they now know: 'Mathematics is there' and 'I can understand the world using math'. All children have tried new challenges, worked with mathematics in new ways and even experienced the joy of doing mathematics in groups/teams.

The purpose of this questionnaire was to identify the effectiveness of teaching non-formal mathematics to younger students through the work – exhibits, workshops, etc. – of math museums around Europe. Based on the analysis conveyed from these four Math Museums (Fermat Science, MMACA, Mathematikum, and Navet Science Center), it can be concluded that the work of the museums is of high importance since it influences, inspires, and encourages the young generation of students to grow up having a great devotion towards mathematics. The playful and engaging exhibits of math museums has emboldened the recreaMATHS project in developing tailor-made e-books as well as 3D printed exhibits inspired by the ones of math museums. By sharing the Math Museums' valuable input, we hope educators understand the greatness of teaching mathematics from an early age as well as provide them with various ideas of how to incorporate mathematics in their day-to-day kindergarten basis.

2.4 Understanding Mathematics as a socio-cultural product; the linkage of Mathematics with the European Cultural Heritage

The European Mathematical Society was established in 1991. Its main objective was to take advantage of the more general establishment of a European community, politically, economically and culturally, to federate initiatives and projects capable of promoting mathematics on the European territory and beyond. But, as in other fields, arguments were based not on

the opportunities and needs of the present (or the future) but instead on a recourse to the history of mathematics which sometimes resurfaced to justify this gathering. According to these arguments, mathematics is basically a creation particularly European. This view is supported by a commonplace account of the development of mathematics which, in its full form, can be summarised as follows: Born in Greek antiquity and forgotten in the dark obscurantism of the Middle Ages, mathematics was created a second time in Western Europe in the 18th century by Galileo, Descartes, Newton, and Leibniz; gradually showing its effectiveness thanks to the development of all the possible applications, and would therefore spread spontaneously over the whole planet.

For most contemporary scholars of the history of science, this account is considered misleading because it neglects the crucial contributions and autonomous thinking of cultures that developed outside the geographical territory of Europe: for example, China, Islamic countries. The mathematicians of these civilizations are presented as mere transmitters of European knowledge. Descartes' generation drew on a vast heritage that was not limited to the sources of classical antiquity. The relationship between mathematics and economic life was important, and multiple, long before aerodynamic modelling or contemporary medical statistics: commerce, architecture, demography, fortifications, thus justified or promoted mathematical development before the 18th century.

The history of mathematics occupies an important place in the dissemination of mathematics or more generally in reflection on this field, whether philosophical, economic or educational. History by force is simplified, just as the mathematics that is popularised. The nature of the simplifications, the rhythm of the overall story presented, the emphasis on one aspect or another, all reflect and simplify the choices made by the

community that elaborates and orders the basic elements. In this case, it is a certain European identity that is forged or activated.

Mediators use this European heritage to develop and promote mathematics. European museums are investing in it, staging it, updating it, sharing it through exhibitions and workshops.

The MMACA presents Leonardo da Vinci (Italian painter, visionary inventor, architect and theoretician) through his work "Vitruvian Man" which shows us the ideal proportions of Man. The body, inscribed in a circle with its navel as the centre, the square with the genitals as the centre. Leonardo da Vinci is referring here to the measurements of length used by the architects of the time. These measures follow a progression linked to the Fibonacci sequence (Italian mathematician); the ratio of one term of this sequence to the previous one approaches the golden ratio as one advance in this sequence.

Another activity at MMACA is the collaborative construction of a dome 1 metre high and 4 to 5 metres in diameter, where the pieces are assembled and form particular geometric patterns to reproduce Leonardo da Vinci's dome.

The European cultural heritage is the medium for the transmission of current mathematics.

Another example is their Eratosthenes room (Greek astronomer, geographer, philosopher and mathematician) with a pole explaining his method for calculating the size of the earth. Manipulation is ubiquitous and a globe and coin are available to reproduce its method.

Source: la-cupola-di-leonardo-da-vinci Mmaca

The Mathematikum takes us on a trip with the graphs of Euler (Swiss mathematician and physicist), children and adults manipulate and try to find a path that passes through all the edges only once, between brainteasers and maths, there is something for everyone. We also find Leonardo da Vinci with his bridge built with small wooden pieces. Pythagoras (Greek philosopher, mathematician, and scientist) with his monochord invented in the 6th century B.C. where visitors can discover fractions and deepen their musical knowledge with the Pythagorean scale.

Source: Euler's Trails

Our European heritage of sculptors and painters is also not to be neglected, many museums are inspired by them for workshops. Geometric shapes and optical illusions are all examples of media that can be animated using mathematical techniques.

Fermat Science also uses this heritage for its workshops. For example, in its "Maths and Arts" section. Vasarely (a Hungarian artist who became a French citizen) shows us with curved and straight lines how to make objects appear without drawing their contours. Pierre de Fermat (French mathematician) invites us to take mathematical challenges combining maths and heritage in his home town, Beaumont de Lomagne. Alan Turing (British mathematician and cryptologist) introduces us to cryptography in a workshop where secret codes will no longer hold any secrets for you.

Source: Fermat Science

The richness of Europe's cultural heritage is a real gold mine for mathematics museums. They can use the work of a mathematician, a monument, a movement, or even a city, as they wish, and transcribe it into a workshop or an exhibition. European museums have understood this and all of them bring it to life, use it, and of course transmit it.

Chapter 3

Adoption of digitized methodologies that appeal to kindergarten educators to broaden and enrich math experiences in kindergarten classrooms

Mathematics, often perceived as too abstract and detached from everyday life, is a nightmare for a growing number of students who, subsequently, reject it. According to Michel Broué, former director of the Institut Henri-Poincaré, "school has turned maths into a science for fools, made up of rote learning of techniques and application of abstract rules without knowing why", whereas this discipline, again according to Michel Broué, is "the very place of imagination, intuition and aesthetics" (Le Point Hors-Série, 2017).

This dislike of mathematics is far from irreversible. Nevertheless, in order to reverse this disposition, it seems necessary to adopt new teaching methods and devices that no longer lead to discouragement, in order to make mathematics a tool for understanding and apprehending the world around us better.

To face this challenge, educators have developed several methods, the main ones being:

- **The Singapore method**

When the city-state of Singapore gained independence in 1965, it equipped its schools with textbooks from Western countries that covered different mathematical researches and pedagogies (such as, those of Bruner, Polya and Montessori). Making mathematics and science learning a national priority in the early 1980s, teachers were commissioned to produce a synthesis of the most effective methods. Over the next fifteen years, the

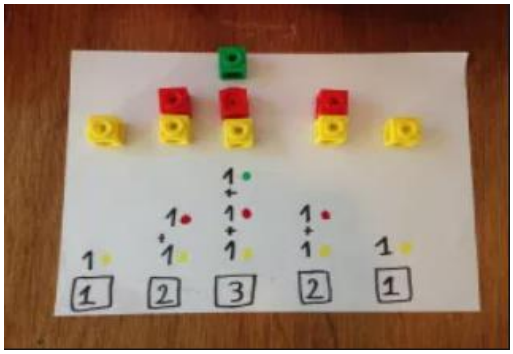
developed method continued to be tested and adjusted. It is now an integral part of the country's education system from kindergarten to primary school.

Indeed, the starting point of this method is that one should not learn mathematics but, above all, understand it. In order to memorise it, it is necessary to understand the meaning and usefulness of the formulae first. Learning them by heart is, in a way, counterproductive. It certainly enables us to apply them mechanically at first, but inhibits us from remembering and applying them again at a later stage.

The method is progressive in the sense that it starts with simple concepts that can be illustrated with pictures. As one progresses, the concepts become more complex and abstract. Learners must be able to construct and explain logical reasoning on their own.

The Singapore Method is based on three fundamental axes:

- Modelling, which consists of getting the learner to construct a plan, a representation of the problem.
- The "concrete-imaged-abstract" approach, which consists of using everyday situations, cubes or other objects to be manipulated and then representing them using circles, bars or dots and finally writing them down using numbers and symbols.
- Verbalization allows the student to explain how they will solve the problem by describing each step of the reasoning.



Source : https://instructionetactiviteenfamille.com/methode-de-singapour-en-maternelle-et-cp/

- **The Assimil method**

This method was created in 1929 by a French language teaching publisher. It is essentially based on intuitive assimilation and thirty minutes of daily work.

The principle is simple: wherever the students are, they can read or listen to formulas and theorems, which allows him to become familiar with the language of mathematics. After each lesson, the students have to practice for thirty minutes. Less than that would be ineffective but more than that would seem like a chore.

Each Assimil book contains between 60 and 150 lessons. Every 5 to 6 chapters an explanatory lesson is proposed in order to repeat the essential points studied in the previous lessons.

- **The Berlitz Method**

The aim of this method, created in 1878, is to enable the learner to think concretely about mathematics and to love it. Learning can take place in two different ways:

- Individually, so that the learner has the choice of the volume and time of his mathematics course(s) during the week.
- In groups of three learners of the same level who have the same objectives, which can limit some biases such as inferiority complexes that can lead to blockages.

Courses, whether individual or group, must take place in a Berlitz-affiliated institution.

- **The Kumon Method**

This method was developed by the Japanese mathematician Toru Kumon in 1954 in order to improve his son's performance in mathematics at his wife's request.

The basic principle is that everyone has an untapped potential that can be revealed. This is achieved by increasing the learner's confidence in themselves and their skills through self-learning.

The method proceeds in stages, i.e., each learner integrates a notion, one after the other. To validate or not the learning of a notion, the learner takes a test where their speed of execution is examined.

The principle underlying this method is to respect the learning pace of each child by placing them in front of a concrete example, so that they can form a mental image that can be generalized and applied to other operations.

Source: https://canal9.ch/apprentissage-kumon-une-methode-pour-developper-lautonomie-et-lamour-des-mathematiques/

3.1 Digitized methodologies to teach Mathematics in Kindergarten

In order to bring mathematics to life, the introduction of a stimulating teaching method that establishes the link with the surrounding world has been deemed necessary. We have seen that other forms of didactics have been developed to meet this objective. However, the emergence of new technologies, and their popularization, have transformed our social practices and our understanding of the world. Moreover, it cannot be overlooked that these new technologies have also had a direct impact on the evolution of mathematical sciences and their practice.

This transformation, although rapid, has been through a lot of stages. Indeed, the enrichment of mathematics learning was initially only envisaged through calculators and professional software such as Excel. "In basic schooling, it is mainly calculators, spreadsheets and dynamic geometry software, as well as microworlds such as Logo" (Artigue, 2011). It cannot be denied that the contribution of these technologies has largely contributed to developing "the possibilities of experimentation, visualisation and simulation; they have modified the relationship to calculation and to geometric figures. They have brought school mathematics closer to the outside world by making it possible to deal with more complex data and more realistic problems" (Artigue, 2011). Nevertheless, it is unfortunate that this contribution has remained particularly underrepresented in basic schooling, and more particularly in kindergarten, due to the lack of training in these technologies for teachers.

More recently, the emergence of the internet and mobile technologies has opened up a new field of research for mathematics learning, both for students and teachers (distance education and training, free access to a wide variety of resources, creation of communities of learners and teachers

among other new possibilities). "It is particularly important to take advantage of these possibilities for mathematics education, especially since it seems that their integration does not pose the same difficulties as the above-mentioned technologies, as they do not affect practices in a similar way" (NMC Horizon Report, 2017).

There needs to be an investment in the field of digital technologies in order to develop tools and even methods that would facilitate the learning of mathematics from kindergarten. "Quality mathematics education for all cannot be achieved without the production of quality resources: resources for students and resources for teachers".

Technology in the classroom has come a long way. It is now more accessible than ever to create interactive lessons, implement participatory learning projects, provide personalised learning and organise classroom activities.

To date, there is a great number of methods and tools available in this regard, some of which are presented in the following non-exhaustive list:

- **Virtual exhibitions and animations**

Today, many museums such as the Palais de la Découverte in Paris, the Maison des mathématiques et de l'informatique in Lyon or the Maison des mathématiques in Belgium offer virtual exhibitions and animations that present mathematics in a fun and interactive way. However, very few are designed for kindergarten learners or even 6/7-year-old children. Nevertheless, they can be a support for teachers, a source of inspiration to create adapted workshops for their students. In addition, some institutions also offer online resources for teachers (files, teaching sheets or activities that can often be downloaded).

Source: https://edu.cospaces.io/PZX-BHK

- **Smartboards**

Interactive and connected whiteboards can replace whiteboards in most classrooms. These boards offer students a much more enriching, fun and interactive experience. They can be used to display videos, photos, drawings and handwritten notes, and thus allow students to approach mathematics from different angles.

Source: https://www.blendspace.com/lessons/XHr58JHuyASScQ/representation-in-the-classroom

- **Online games and apps**

A recent study published in the journal Pediatrics by Shayl Griffith and colleagues found that the use of online games and apps was linked to higher mathematical ability in early childhood. However, it is important to remember that the teacher needs to think beforehand about how the game or application fits into the curriculum. According to Theresa Wills, an assistant professor of mathematics education at George Mason University in

Fairfax, "It's a great data point to see where kids are at and then teachers can use it to develop appropriate interventions for students who need more support".

Source: http://www.mathblaster.com/

- **Badging and gamification**

Badging and gamification are motivational tools that reward the learner through the use of digital badges. After reaching a certain level of difficulty, the learner is awarded with digital badges. Thus, the acquisition of new badges forms a collection and transforms the learning process into a gamified process that is supposed to generate enthusiasm in the learner and encourage them to continue their efforts. However, for this method to be effective, it is necessary that the game/learning is linked to a specific pedagogical objective. Furthermore, while this method can create a form of

emulation between learners, it can also prevent their cooperation and collaboration by creating rivalries.

The different technologies that can be used can all help to identify areas where learners are having difficulties and allow the lesson, or the pace of the lesson, to be adapted accordingly. In addition, they encourage interactivity and can therefore reduce the passivity of some learners. Furthermore, we all live in a digital and technological world. It is therefore essential to have access to and manipulate these tools from an early age "to generate a deeper understanding of the digital environment, allowing intuitive adaptation to new contexts and co-creation of content with others".

Despite all the benefits that digital methods can bring, it must be admitted that they can also be a source of disadvantages. To be more specific, the first con is undoubtedly the fact that there is a certain inequality of access to digital technologies due to differences in infrastructure and equipment between different territories and between individuals. Many learners, whether in the classroom or at home, still do not have the necessary equipment to access digital content. Furthermore, and we will come back to this in more detail in our third part, not all teachers are trained and therefore do not master these technologies. Thus, the contribution of digital technologies to learning is, in these cases, non-existent.

It should also be noted that digital technologies may distract some learners, as their attention may be captured by other features available with these technologies. It is therefore imperative to counter this effect by making the course particularly lively and interactive. There are also concerns that these technologies are an obstacle to the development of verbal communication. Again, it is essential that these technologies are seen as tools, not as ends in themselves, promoting dynamism and interactions between learners and

between learner and teacher. The keystone of school learning is, and will remain, the relationship between teacher and student.

Finally, as Rémi Brissiaud, a researcher at the Laboratoire Paragraphe, points out, "we should be careful about the race to develop digital resources. This rush risks being counterproductive because a question arises: is it reasonable today to mobilise so much goodwill, energy and funding to develop digital resources when, without this essential resource of reference to a common vocabulary to describe children's digital progress, serious didactic errors are likely to be made?" (Brissiaud, 2014).

3.2 Employing digitized tools and technological innovation for the creation of educational material by the kindergarten educators with the aim to reinforce mathematical learning in kindergartens

The use of digital technology is becoming more and more important in our daily lives and its integration into teaching practices is evident. The various technological innovations have led to pedagogical innovations by rethinking the way of teaching, i.e., the space given to interactions, autonomy and collaboration. Digitalisation should be seen as a tool that teachers can use to create their own teaching materials. It is not (or should not be) a substitute for the role of the teacher and their relationship with the learner.

The following is a list of different digital tools and innovations that can be used as complementary materials to support mathematics learning:

- **Course platforms**

The courses offered on the Khan Academy platform

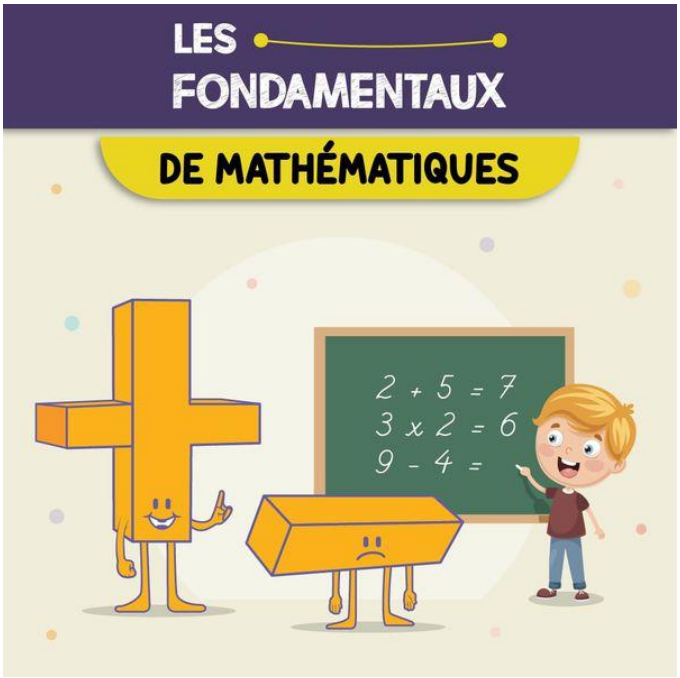
(https://www.khanacademy.org/math) consist of videos that illustrate the explanation of a concept or the solution of a problem, i.e., animations

illustrate what is presented orally. The learner can then practise through a series of application exercises before taking the test that will allow to access the next course. The platform offers support for teachers as well. More precisely, it provides a monitoring tool that allows them to access the students' achievements, the videos they have watched or to see the time they have spent on the platform. It should be noted that the content made available on this platform is completely free and without advertisements (the platform is financed by donations).

- **Educational video platforms**

The Lumni platform, provided by the French public broadcasting system (i.e., free of charge and without advertisements), makes available a number of files, including one on the fundamentals of mathematics (https://www.lumni.fr/dossier/les-fondamentaux-de-mathematiques). The latter is divided into several sections (geometry, numbers and calculation, units of measurement, etc.) which deal with the various concepts to be acquired through short videos. This platform enables learners to consolidate the elements studied in class and teachers to have access to expert resources.

Another example is the Canopé network, whose mission is to strengthen the action of the educational community in favour of student success. Through several platforms, the network offers resources for teachers. The "Les fondamentaux" platform (https://lesfondamentaux.reseau-canope.fr/discipline/mathematiques) in particular publishes a series of short, entertaining videos that can enrich a lesson by visually illustrating it.



- **Edutainment sites**

The Mathador site (https://www.mathador.fr), which is published by the Canopé network, offers two digital versions of its mental arithmetic game, which is designed for children aged 7 and above and uses multi-sided dice (10, 20, 6, etc.). "Solo" takes the form of a 30-level course in which the mental arithmetic operations become more complex as they progress. "Chrono" allows players to play alone or in pairs on the same screen as well as in a network against friends or random players. The player has 3 minutes to solve as many operations as possible, the more difficult the calculations, the more points they will get. Schools can take advantage of a "Mathador

Classe" subscription that gives each pupil access to the two games, allows the class to take part in the Mathador competition (one test per week), and gives the teacher an interface where they can monitor the progress of their pupils.

The Matheros platform (https://matheros.fr/) is aimed at children from the age of 6 and also focuses on mental arithmetic. However, it is designed to be used with the whole class, as the teacher selects the exercises that the pupils should work on, either at home or in class. Each student plays a superhero whose goal is to liberate a city. The superhero will acquire new powers, i.e., new skills, depending on the student's progress in mental arithmetic. The skill belts are obtained by going through four different stages: learning, practising, playing and finally validating the knowledge. This platform allows the teacher to follow the progress of each student and to adapt the following exercises according to their results.

The Number Race (http://thenumberrace.com/nr/home.php) is a free downloadable game for children aged 4 and above that teaches the basic concepts of numbers and arithmetic. This game is designed specifically for

children with dyscalculia and reinforces the different mental imprints of the number (digit, word, quantity, position) and therefore its meaning.

The software is licensed under the Gcompris free license (https://gcompris.net/index-fr.html) and is aimed at children from the age of two. It is translated into 57 languages. Many edutainment activities related to mathematics are available. A teacher's interface also allows the selection of games adapted to the level of the pupils or the material available.

Math Mathews is an application that helps someone learn multiplication tables. It follows the adventures of a brave pirate who, under a curse, is transformed into an octopus. In order to return Math Mathews to his original form, the child has to solve multiplications within a given time frame. As they progress, they will earn badges for each of the tables from 1 to 9.

- **Innovative applications**

Dragonbox Numbers and Big Numbers (https://dragonbox.com/products) are based on the Cuisenaire short rulers' method, which represents the numbers from 1 to 10 in the form of bars. In these two applications, the

short rulers have become Noums, a kind of friendly little monster that allows the learner to be introduced to numbers as a relationship between quantities. The learner can see how the Noum is made up (how many units), cut it up and thus give two smaller Noums. They can also make the Noums eat up each other to make a larger Noum. The three basic actions of problem solving (putting together, breaking down, and comparing) are thus learned here in a playful way and the association of gestures and concepts allows the cognitive processes to take root more effectively, thanks to the sensory dimension.

Edoki develops applications designed by Montessori teachers (https://montessori.edokiacademy.com/fr/catalogue/maths/) and are available in 8 different languages. The math-themed applications aim to reinforce counting, to teach how to write numbers and memorise their names, understand the concept of zero, and to introduce the first operations. Edoki has also developed a broader application, "La Maternelle Montessori", which is accessible via a subscription and includes a myriad of educational mini-games.

- **Applications involving the manipulation of solid objects**

Marbotic (https://www.marbotic.com/smart-numbers/) has developed a method of learning numbers for children between 3 and 6 years old inspired by the Montessori method, which advocates the manipulation of physical objects to promote the acquisition of abstract concepts. Here, we can find coloured wooden pieces that can be interacted with on an application. Children enhance their motor skills while discovering numbers through their different representations (sound, image, calculation, etc.).

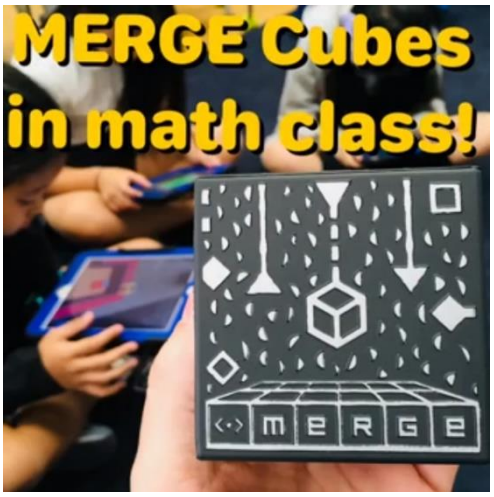
Osmo (https://www.playosmo.com/fr/shopping/games/numbers/) has developed a base and a sensor that attaches to the mobile device and detects the game pieces in front of it. The learner's manipulations of the pieces are transposed onto the device's screen, which helps to improve concentration and fine motor skills in addition to cognitive abilities.

- **3D modelling**

3D modelling is of increasing interest to schools because it encourages transdisciplinary. It allows the initiation of projects that combine mathematics, technology and the plastic arts.

- **Augmented reality**

Augmented reality is a process that makes it possible to add virtual content to the real world, which can be viewed using a digital device. The advantage of augmented reality in learning is that learners, immersed in a dynamic environment, discover concepts in action and make them their own. The Merge Cube (https://mergeedu.com/cube) is one of these new solutions and is particularly accessible to young children. Through the "Frame" application, children have access to different written or illustrated mathematical challenges that they have to solve with the Merge Cube. Once they have solved all the challenges on one side of the cube, the cube is colored and the next challenges on the other sides must be completed.

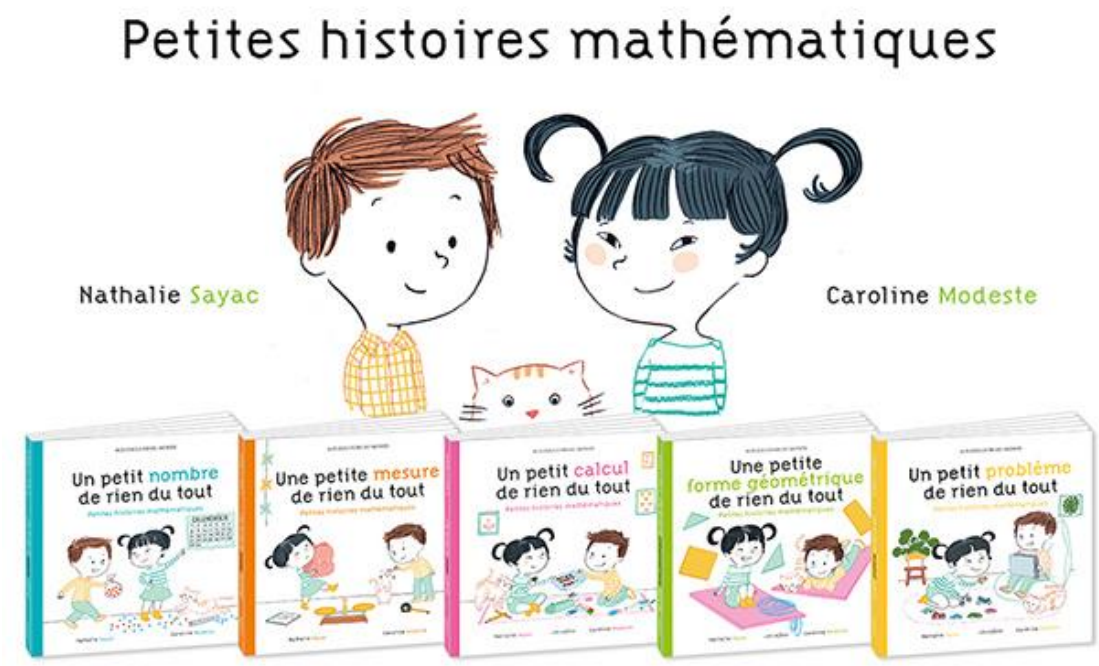


- **Books**

Books have always been an excellent way of teaching complex concepts to young children, such as those that mathematics deal with. Some publishers, driven by the need to make mathematical concepts ever more comprehensible, have come up with original initiatives that we can separate according to three different approaches.

Firstly, we observe a narrative set up around mathematical objects allowing them to be better understood. This is the case with Petit cube chez les tout ronds, published by Mijade, in which the heroes are geometric objects with mathematical characteristics; or Raconte à ta façon, published by Flammarion, in which the characters of famous tales are represented by different geometric shapes that the children manipulate to reinvent the story.

The second category of books features children confronted with mathematical problems in everyday situations, for example in the collection Petites histoires Mathématiques published by Circonflexe. Solving these problems allows children to approach the first concepts of mathematics in a playful way, but also to develop their mathematical thinking.



Finally, the third category includes books that offer the opportunity to students to discover mathematics through the prism of the history of science and its illustrious characters. The aim of these stories is to provide role models in the field of sciences and render some concepts more accessible by demystifying these figures.

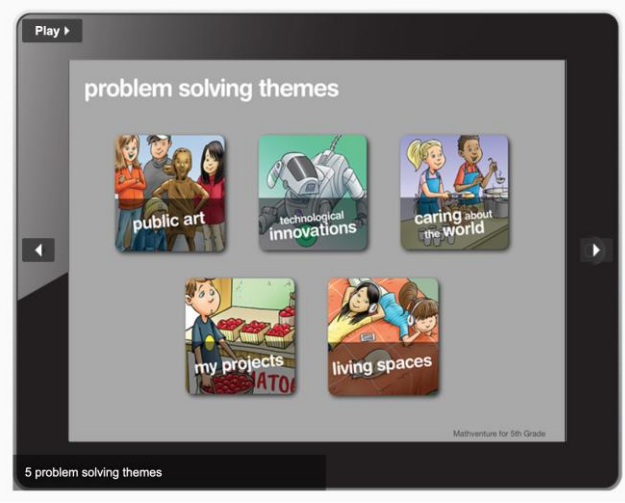
- **E-books (or electronic books)**

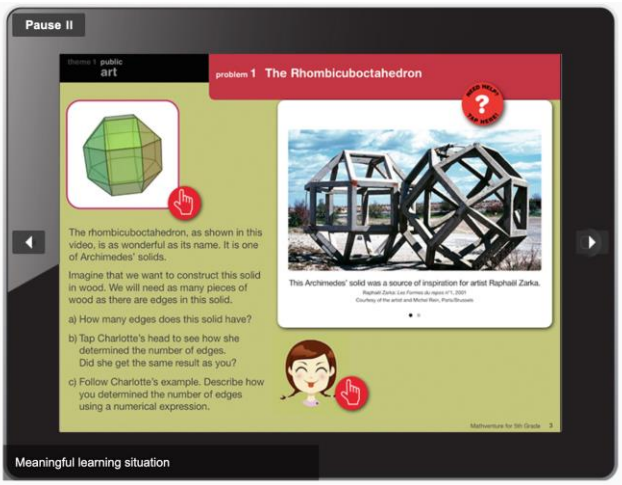
The release of the iPad by Apple in 2011, and then tablets in general, has changed the age at which electronic materials are accessed. With electronic screens now approaching the format of a book, educational content publishers have seized the potential of such devices and have begun to adapt their works for digital libraries.

Of all the varieties of ebooks, the EPUB fixed layout is the one most frequently chosen by publishers because of its cost of implementation. In fact, they are most often the adaptation of paper books, i.e., no additional work is done on the layout or on adding interactive or edutainment content.

They are therefore the most economical and simplest solution for publishers specialising in illustrated books or books with complex layouts such as school textbooks.

However, new players have appeared on the market and are trying to develop these practices through specific work on the layout and the integration of interactive content in school textbooks. Among those who are taking this approach are Nathan – Mon cahier Maternelle, which offers a set of interactive and audio activities to introduce children to numbering and spatial orientation, with a reward system for each success; and Baobab Education in Canada, which has invested in the creation and implementation of a collection dedicated to mathematics for grades 3, 4 and 5. The books have been praised by teachers and awarded for their innovation and interactivity for the explanation of mathematical concepts.



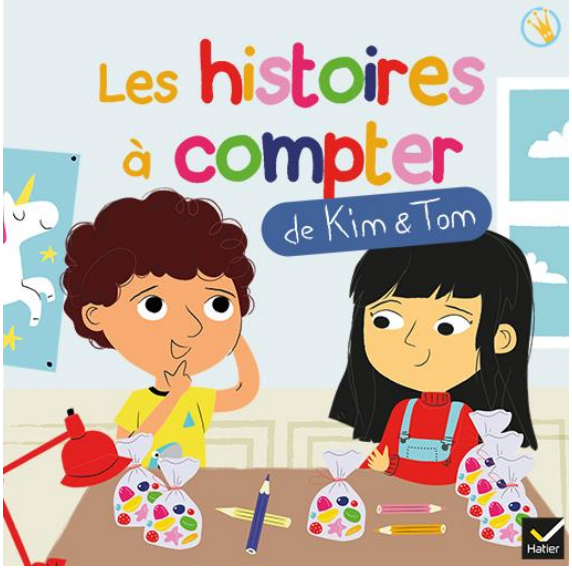


- **Audiobooks**

Lunii, a new publisher that has developed its own material support, "Ma Fabrique à Histoires" for its audio and interactive books, has also recently taken an interest in educational and scientific content.

In the series of books "I calculate Step by Step with Kim and Tom", everyday situations are presented. At the end of each chapter, a mathematical question is asked and, to continue the story, the children must choose a correct answer.

In the series of books "I calculate Step by Step with Kim and Tom", everyday situations are presented. At the end of each chapter, a mathematical question is asked and, to continue the story, the children must choose a correct answer.



3.3 Using technological innovation for the training of kindergarten educators

Technological developments and their integration into the field of education require teachers to learn new skills. Indeed, how can knowledge be transmitted with the help of tools whose operation is not mastered? Moreover, the issue of digital use reflects disparate situations both in terms of the skills of different teachers and in terms of the resources allocated for training and equipment. So how can we systematise the use of digital

technologies in teaching methods, but also how can we harmonise practices so that their integration, or not, in education does not become an additional source of social inequality? This is a challenge that initial and in-service teacher training must now meet.

A recent study shows that two out of five education professionals have low or very low skills in new technologies. "Education today and in the future depends on the development of work communities in which diverse competences complement each other and the skills, attitudes, and knowledge of teaching professionals can be enhanced by professional development. Increasing the use of digital technologies in pedagogical practices should provide teaching professionals with recurring opportunities to develop their skills" (Hamalainen et al., 2020). Institutions, therefore, have a central role in offering more training courses that would enable teachers to acquire real techno-pedagogical expertise.

Tools for better teacher training already exist. Some of them may serve as a basis for thinking about new ones, more in line with the needs expressed by kindergarten teachers:

- **M@gistère digital platform**

Several training courses relating to the integration of digital technologies into learning are available: "Digital games for learning", "Supporting pupils' work with digital technology", "Using the NIC in teaching", "Teaching with digital technology: the digital kit", "MOOC Tablet 2", and "Digital technology and updated curricula - primary level". As regards learning mathematics in kindergarten, the available material is not as extensive. To be more specific, only three courses are offered: "Giving meaning to 'counting' in kindergarten", "The Construction of Numbers in the Middle and Upper Kindergarten" and "1,2,3... Building numbers". It should be noted that no

training program directly links the teaching of mathematics in kindergarten with digital technologies.

The principle of e-learning may at first glance be attractive, as it allows the teacher to organise their lesson as they wish. However, many see it as an additional burden, a tedious task to be carried out outside of one's actual working time. Furthermore, as far as the M@gistère platform is concerned, the ergonomics are often questioned and do not facilitate immersion in the training. Moreover, there is little room for sharing and exchange.

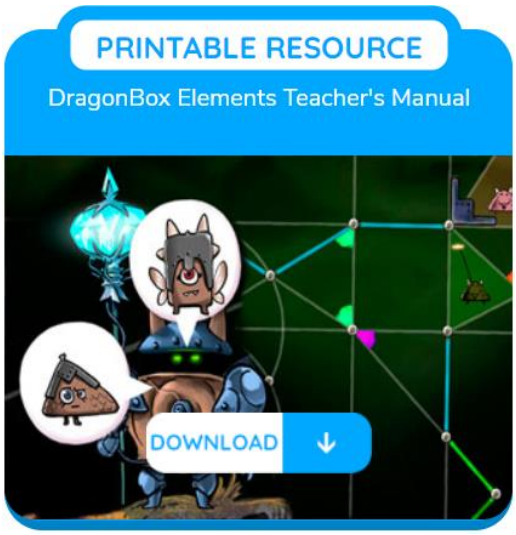
- **Enhanced e-books**

Many textbooks in the form of digital books are available for teachers. They are most often adaptations of the same content in paper form. The Canopé network offers a wide range of books for kindergarten teachers on its website, including the very comprehensive "Enseigner les mathématiques en maternelle" (https://www.reseau-canope.fr/notice/enseigner-les-mathematiques-en-maternelle.html).

- **The Mathematics laboratories**

Although the "labomaths" were not initially intended to be implemented for kindergarten teachers, the approach seems interesting enough to be mentioned.

The "labomaths" have been developed to contribute to the professional development of teachers in teams. They are a place for ongoing training and reflection on the subject matter, didactics and pedagogy. They encourage interaction between teachers and allow the presentation of innovative practices and collaborative problem solving. The "labomaths" aim to reach out to their territory and establish relationships with partner establishments. They are also a place for producing resources (teaching resources, articles with disciplinary, didactic or pedagogical content, production of videos, sharing of content produced during events or sharing of experiences in various multimedia forms) which can be shared.



The use of new technologies most often requires technical skills and knowledge. However, there are digital solutions that provide teachers with both a tool and a method, i.e., how to implement the integration of this tool into the learning programme. This is the case with Marbotic and Dragonbox, both of which, as we saw earlier, are inspired by the Montessori method. Both offer teachers a pedagogical booklet that explains how the activities that can be carried out with their tools fit into the learning programme established for the class and guides them through this integration. For example, Marbotic, for "10 fingers", explains in a preamble what skills are targeted in the kindergarten curriculum and how its application meets this

requirement. A programme is then proposed in which, for each session, the content of the session, what the pupil learns and the conditions required for the proper conduct of the session are indicated. Dragonbox has a specific access for teachers on its website (https://dragonbox.com/educators) and provides various downloadable teaching guides for its games.

Chapter 4

Alternative pedagogical methodologies and synchronous interdisciplinary best-practices to approaching simple math concepts and reasoning for preschool ages

4.1 What is interdisciplinary learning and why is it important in preschool?

With the term interdisciplinary in education and training pedagogies we describe the use of methods and insights of several disciplines or fields of study. The adjective interdisciplinary is most often used in educational circles when researchers from two or more disciplines pool their approaches and modify them so that they are better suited to the specific situation, including the case of the team-taught course where students or children are required to understand a given subject in terms of multiple traditional disciplines.

Interdisciplinary teaching and learning are maximized when several professionals and experts from different disciplines work together to serve a common purpose to help students integrate and make the connections between different disciplines or subject areas, along with their specific perspectives. By doing so, students can apply the knowledge gained in one discipline to another discipline. Repeated exposure to interdisciplinary thought can help learners develop more advanced knowledge, enhance critical thinking ability and metacognitive skills, and increase the understanding of the relations among perspectives derived from different disciplines.

It is common in scientific research, that the provision of choice-making is an indicator of developmentally appropriate practice for young children

with and without disabilities; however, there is little empirical evidence regarding the rate of delivery of choices within the preschool classroom. The introduction of intervention strategies by a team, that is based on an interdisciplinary classroom, also is not well documented, which makes it necessary to increment the exposure of children to this kind of strategy.

The new curriculum for preschool education allows the engagement of the young pupil in as many experiential fields, through learning experiences and this "allows the interdisciplinary integrated approach of the suggested contents and provides freedom to the teacher in planning the daily activity with young pupils. Therefore, the interdisciplinary approach is required even by the curriculum for preschool education, as a logic requirement of modeling the young pupil and as a natural modality of action with the contents of several experiential fields" (Dinuță, 2015).

Since the last few years, **interdisciplinary learning has been an issue but a necessity in preschool education**, yet the traditional nature of many institutions and kindergarten has barriers that in many ways discourage or prevent such activities from happening. For example, teaching in teams from various disciplines to children is commonplace, but there are often tensions when combining experts from multiple fields, and this always helps to consider chemistry and fit when talking about team building.

Moreover, new and emerging technology skills are an essential part of standard curricula in many disciplines today because technology shifts occur at a rapid rate that it is virtually impossible to keep up with them through the traditional learning strategies. (Lorenzen-Huber et al., 2010; Loewer, 2012).

On the other hand, one of the clearest advantages for students and kindergarten children is the reality that multiple instructors enrich a

student's learning experience through diversity exposure and multiple points of view.

For instance, we have loads of examples of themes that cross over disciplinary boundaries in literature, art, and history or science and mathematics. The implementation of Science, Technology, Engineering, and Mathematics (STEM) at school or preschool is one of the challenges of education in the twenty-first century for many countries (Sanders 2008), especially concerning the development of critical thinking during argumentative interactions. The literature on scientific activities in education and in particular in early childhood is relatively recent (Impedovo et al. 2017). Most of the studies about how to implement and analyse STEM in education deal with elementary, middle, or high school (Smyrnaiou et al. 2015). Regardless of the specific STEM field, argumentation is in fact a transversal process of knowledge construction about the natural world (Erduran and Jiménez-Aleixandre 2008). It is also a process of critical reasoning that is important for the development of young citizens (Schwarz and Baker 2017). As the mind is argumentative by nature (Moshman 2004), young children are naturally inclined to explore the environment and to ask questions about scientific phenomena (Danish and Enyedy 2015; Ravanis 1994).

Another aspect is the student's high motivation due to interesting topics and the interest that these topics can create in them. As a result, one of the most effective ways of presenting the content is often to connect activities in life experiences, giving an authentic purpose for the learning and connecting it to a real-world context. Consequently, the learning becomes meaningful, purposeful and deeper resulting in learning experiences that stay with the student for a lifetime.

As students look across disciplinary boundaries, critical thinking skills and metacognitive skills are used and developed to consider other points of view. Worthwhile topics of research can fill the gaps between the traditional disciplines. Children also begin to compare and contrast concepts across subject areas, which helps them develop their personal inner world and thoughts.

Moreover, students begin to consolidate learning by synthesizing ideas from many perspectives and consider an alternative way of acquiring knowledge. Of course, the consequence of the repeated exposure and exploration of topics across a range of subject boundaries motivates students to pursue new knowledge in different subject areas, which results in transferable skills of critical thinking, synthesis and research. These skills are developed and apply to future learning experiences, which will be applicable not only in the specific topic handled, but also to transversal subjects across disciplines. Therefore, interdisciplinary knowledge and application of different disciplines can lead to greater creativity.

4.2 Approaching Mathematics in Kindergarten as a multifaceted and interdisciplinary field

With raising research results the knowledge of how children develop mathematical skills is gaining consistency. It is recognised that "children require significant amounts of time to develop the foundational mathematical skills and understandings." (NRC, 2009, p. 124). It is therefore crucial that enough time is dedicated to mathematics in preschool programs so that children develop foundational mathematical skills and knowledge. But it should not only focus on the more formal parts of mathematics instruction and discussions but also include non-formal education like exploring, creating, and playing. (NRC, 2009, p. 124)

The potential of everyday activities such as cooking, playing with mathematical shapes and telling the time is recognised and harnessed in order to empower the children learning program. Also, everyday experiences using mathematical words and phrases is one of the key elements for children to induce them to talk about their mathematical thinking.

Essential is the children's engagement in what is interesting and relevant to them and the experiences that show the usefulness of mathematics for solving everyday problems. Children's eagerness to participate in everyday activities such as cooking (Vandermaas-Peeler et al, 2012), or shopping is an effective way of fostering a positive disposition. As a result of her study of the number sense of 4-year-old children, Dunphy (2006) concluded that the ways in which children are engaged with mathematics, how they view mathematics, and the contexts in which mathematics are presented to them are what shape their dispositions towards mathematics. In the same study, children with a positive disposition also demonstrated a strong number sense. For instance, young children starting school may already have developed a liking or enthusiasm for numbers, based on experiences during the preschool period, i.e., their disposition towards number is already developing (Dunphy, 2006).

Effective teachers use a variety of 'worthwhile mathematical tasks' and help learners 'make connections' across mathematics, between different solution paths in problem-solving, and between mathematics and everyday life. Effective teachers of mathematics carefully choose 'tools and representations' to stimulate and support learners' thinking. From a teaching and learning perspective, projects are a valuable approach to organising mathematical activities for young children (Katz & Chard, 2000; Ginsburg & Golbeck, 2004). Listed in the table below, is a variety of possible applications of mathematics in everyday life and in activities that can be developed in early childhood.

FIELD	DESCRIPTION	MATHEMATICAL CONCEPTS
DIGITAL TOOLS	using technology is an increasingly important avenue of learning and expression for children. For example, Kalas (2010) describes children's engagement with technology and digital tools and exploration of direction and location	• Exploring spatial concepts • Develop the language of spatial relations (e.g., besides, towards, short / shortest) • Develop algorithmic thinking (processes or rules for calculating)
COOKING	A study of water (adapted from Dixon, 2001)	• Document processes through diagrams, drawings, charts, photographs, data and models • Explain mathematical processes
	Teaching Practices Making apple sauce (adapted from Ginsburg & Golbeck, 2004)	• Decide how many jars of apple sauce are required • They count, 'read' a pictorial recipe • Discuss the itinerary to the supermarket • Weigh ingredients, compare size, shape, colour, and price of fruits

	The pizza project (adapted from Gallick & Lee, 2009)	• The sequence of making pizza • Estimate, measure and cut circles of paper to represent pizza slices
MUSIC	Shilling (2002) identifies a strong link between the order, timing, beat and rhythm of music and attributes of mathematics such as counting, sequencing and understanding time and order. This enables teachers to make learning both music and mathematics more meaningful for the children (Kim, 1999; McGrath, 2010; Montague-Smith & Price, 2012; Pound, 1999; Shilling, 2002).	• Development of other skills and attitudes that are important for mathematics such as concentration, creativity, perseverance, self-confidence, and sensitivity towards others (Fox & Surtees, 2010) • Develop children's mathematical language and concepts
VISUAL ARTS	Pattern and shape are key features of both the visual arts and mathematics. In the visual arts, children encounter colour, form, texture, pattern and rhythm, and shape (Government of Ireland, 1999c). In mathematics, they discover patterns of number and shape, symmetry, tessellation, and the	• Develop the child's awareness of the visual and spatial qualities in the environment • Enhancing children's ability to apply mathematical knowledge in the environment

	properties of a range of 2D and 3D shapes. Drawings provide a forum in which children's confusions about particular aspects of mathematics can be addressed (e.g., orientation can vary).	• Identifying 2D shapes in fabrics • Symmetry in pictures • Perimeters • Convey their growing awareness of number and quantity • Develops abilities to translate mathematics from one language (verbal) to another (graphic) (Worthington & Carruthers, 2003).
DRAMA	Drama and PE Role-play offers many opportunities for children to engage with mathematical concepts and skills. Story contexts such as 'The Three Little Pigs' can give rise to a range of mathematically-related play, especially if appropriate props are provided to stimulate mathematical thinking (e.g., Pound, 2008).	• Phrases such as 'just enough' (equality), 'not enough' (less than) and 'too many' (greater than) can be used and their meaning explored in the context of the play • Forming groups for games, representing basic processes such as addition or subtraction, by combining or separating groups of children

		• Partitioning of numbers can be explored
SPORT	Participating in swimming or athletics. Very young children can be exposed to mathematical vocabulary through everyday discourses such as swimming lessons (Davies et al, 2012).	• Creating 2D shapes using children's bodies, and discussing the properties of such shapes • Calculating times and distances
MEASUREMENT	Measurement is an important mathematical topic because of its applicability to everyday activity, its connection with other subject areas, and it can serve as the basis of other content areas in mathematics (Clements, 2003).	• Vocabulary for quantity or magnitude of a certain attribute • Comparing two objects • Equality or inequality • Overcoming perceptual cues
NATURE	Observation of plants, vegetables and fruits geometric forms.	• Geometric pattern observation • Develop the child's awareness of the visual and spatial qualities in the environment

4.3 Inclusive Math Approaches

To ensure that children have a rounded and fulfilling educational experience, **an inclusive math approach is a crucial part in achieving this goal**. The opportunity should be given to engage in a wide, balanced and rounded curricular experience that supports all aspects of their development – not only the formal part and content acquisition, but the social, emotional, imaginative, aesthetic, and physical dimensions as well.

Children need to be assisted in using the newly acquired mathematical language in their descriptions and explanations. Good mathematics pedagogy recognises that some children (e.g., children living in disadvantaged circumstances; children who speak a language that is different from the language of instruction) may experience difficulty with problems presented in verbal format and there may be a need to adjust the presentation accordingly (Ginsburg et al, 2006).

Most important is that children play an active role in the development of their knowledge and to be developmentally appropriate and avoid premature formality.

Other contexts in which preschool educators can promote mathematical concepts and language include, for example, playing games, reading books with a mathematical theme, using computers, and constructing objects (e.g., block building).

Just as mathematics is learned in context, so it is used in context to achieve some worthwhile purpose.

Dan Finkel, a Ph.D. in mathematics from the University of Washington, says that mathematical miseducation is one of the most common things and we expect lessons to be about memorization and repetition of disjointed

technical facts. Therefore, children need to learn that mathematics is not about following rules but about playing and exploring[1].

Classrooms are filled with all types of learners and the teachers' role in kindergarten is critical because it will set a base for the learners' future and have an impact on how they will perceive different topics later on. It is also a time to identify certain difficulties and help children strengthen their problem-solving capabilities. Therefore, a kindergarten curriculum must be well thought of and cover different styles of learning steering positivity into topics that have proved to be challenging in later school years for example mathematics. The teacher in kindergarten can stimulate the interest of the pupils towards learning and creativity through different pedagogical approaches. This phase in a child's education is where educators can start understanding their individuality in learning and how they adapt to different activities.

4.3.1 Learning Dys-

All types of "dys" may affect the learning of mathematics but the most common **Math related Specific Learning Disorder (SLD) is Dyscalculia**. Dyscalculia affects a person's ability to understand numbers and learn mathematical facts. Students with this SLD often lose track while counting, mistake numbers during operations, have trouble memorizing and recalling mathematical procedures and rules. Students with Dyscalculia typically perform poorly in tests and become easily overwhelmed and develop math anxiety.

Math related disorders can start to be evident from preschool age according to the article "Specific learning disability in mathematics: a

[1] Five Principles of Extraordinary Math Teaching | Dan Finkel | TEDxRainier, https://www.youtube.com/watch?v=ytVneQUA5-c , retrieved on the 11th February 2021.

comprehensive review” in the Translational Pediatrics Journal. Toddlers may start showing difficulty learning to count, sorting, corresponding numbers to objects, memorizing numbers by hearing them. In order to have the earliest diagnosis it is critic that children are exposed to mathematics as early as possible. The earlier the diagnosis is made, the earlier the children may get help to develop good foundational skills.

Rochelle Kenyon, a consultant and trainer in education and disabilities topics, lists a series of strategies to teach students with math related learning disabilities. The first strategy is to not overload students’ memory and assign tasks in manageable amounts as skills are understood. The premise of recreaMATHS is to shift the emphasis from “numeracy skills” and ‘skill and drill practices' to teaching the ‘language of math’ focusing on the comprehension instead of the fast memorization of math procedures. This strategy profits all types of learners because it proposes an adapted timetable for task assignment. The division of activities in clear, short steps, allow enough time for students with SLD to grasp the concepts. Memorization is not the strong suit of all students, so this method also favors those who need to understand mathematical concepts in order to learn math. It is easier for students with SLD to focus more on logic than memory. Therefore, one of the approaches is to challenge critical thinking to boost problem solving within learners with SLD using real life problems to render mathematics more tangible. Using real-life situations changes the point of view on mathematics making problems functional and applicable to everyday life.

The provision of supervised activities to ensure the good practice of mathematical concepts and the proper application of rules is another approach that can benefits students with SLD that is covered by recreaMATHS for example with the module on 3D modelling. During the supervised activities it is important to give constant feedback to learners and

also have them track their progress. Another strategy that aligns with recreaMATHS approach is the use of manipulatives and technology specially with the help of the 3D modeling module.

Rochelle Kenyon advises to help students visualize math problems by drawing, this is an approach that helps learners to understand better using visual elements to illustrate concepts. She also advices the use of auditory examples by focusing on a multisensory method. The project will be applying this approach in most of its intellectual outputs, for instance mathematical e-books for children between 4 and 5 years old and a version for children between 6 and 7 years old, an entire collection of 12 hands-on plus 2 Virtual interactive Mathematical exhibits for kindergartens and finally 3D modeling with a 3D printer. These intellectual outputs also correspond to the Rochelle Kenyon's strategy of using age-appropriate games as motivational materials. If possible, it is better to make creative, constructive activities, rather than exclusion or competition-based activities.

Distractions and unnecessary information in classes with learners with SLD must be avoided, for this reason cluttered worksheets and too much visual information is not helpful specifically for students with ADD, ADHD and Dyslexia, structuring papers with clearly distinguishable titles and subtitles is a good beginning to have a better structure.

4.3.2 Deaf and Blind

Mathematics is simple, but the language used to explain it is complicated. The difficulty in decoding the language determines the limits of understanding for most of hard of hearing, blind and deaf students, who find themselves, considering it abstruse and complicated, not to like the discipline. The ability to extract relevant information from a text, with respect to the resolution of the same, is influenced by the mode of

presentation of the text. Mathematics is a communication code that, although different from other codes such as language or sign language, can be compared to them.

In an article, from the Journal of Research on Technology in Education (vol.45 no.4) "Teaching Mathematics Vocabulary with an interactive Signing Math Dictionary", Judy Vesel and Tara Robillard documented the **five most common problem areas among deaf students**: words with more than one meanin; technical language; specific words in mathematics; the presence of varied but related forms; and specific abbreviations and symbols. Mathematical language, the simplest mathematical symbols, (+, -, x, :), numbering in base 10, are, for the deaf, an ideal language because each sign, each digit, each rule has a meaning for itself and for the position they occupy. In the early stages of learning, the deaf person seems to have an easier time with mathematics than with language. But when, through language, we have to solve problematic situations, they have enormous difficulties.

Teaching mathematics to a blind person is not entirely different from teaching it to a sighted person, but it presupposes knowledge of the ways in which concepts are formed and the world explored that are typical of the blind. In fact, blind people need a longer time to approach reality, as such is the tactile exploration of objects. In addition, it is important to assess whether what the blind express in words are learned concepts, or are the result of verbalism, repetition of terms empty of their meaning. It is always necessary to follow an order that goes from concrete to abstract and therefore at the basis of the formation of any concept there is the manipulation of reality, then the representation, and finally the symbolization.

When talking about deafness, children and integration, we are talking about 'special' inclusion. Indeed, the quality of inclusion depends to a large

extent on the capacity of the school or classroom to become a supportive community. It requires a high degree of flexibility on the part of the teachers and the pupils themselves in a spirit of mutual support and coeducation. A redefinition of the school context is therefore the main task of the educational institution and should focus on being aware that the hearing and visual deficit only limits the ability to hear and see but does not cause any cognitive damage.

To be able to overcome difficulties and win the attention of the student, it is necessary to aim at effective communication, since the main obstacle to overcome is precisely at the communicative level. Therefore, one of the first fundamental suggestions is to reduce the time of frontal teaching and adopt strategies and modes of explanation that make it easy to understand, considering the specific needs of each deaf and blind student. Moreover, it is preferable to privilege a dialogic style rather than a tutorial one. Also, a kind of peer-to-peer adaptation is needed, during which all, blind, hard of hearing and deaf people should try to cooperate for a good communication strategy.

Specifically, for deaf children in this regard, it is necessary to stop talking in class when one is turned to write or draw on the blackboard; speak in turn, one at a time, and signal with the hand when someone interrupts and intervene in the conversation; touch the child lightly on the arm to call his attention, never suddenly and behind his back; make him participate in everything that happens in class and that he/she may miss.

Listening in a noisy environment is a source of difficulty and stress: for almost all blind, deaf, hard of hearing and students with traditional hearing aids or cochlear implants, the presence of noise means that they cannot refer to the ear canal. Even for deaf students who do not use any aids, background noise can be a source of discomfort. **To improve the ecology**

of the school, making the environment more conducive to listening and/or communicative exchange we make the following suggestions:

- Provide for the presence of deaf and blind students by placing them in a non-noisy classroom, use soundproofing materials (curtains on windows, carpeting, rugs) and anti-noise materials (grommets on chairs, bumpers on doors).
- Verify the appropriateness of using frequency-modulated (FM) systems that improve the signal-to-noise ratio (25 dB).
- Arrange desks in a semicircle or otherwise so that the student can easily see and hear both the teacher and his or her peers.
- In cases where sign language is the student's preferred language, ensure that the communication assistant or teacher has an easily visible accommodation.
- If the child uses a cochlear implant or traditional hearing aid, the teacher should familiarize him or herself with the aid.
- Use good lighting, making sure the light source is not glaring.
- In order for the deaf person to lip-read well, the optimal speaking distance should not exceed one and a half meters, and the speed of speech should be moderate. Lip-reading is based on correct pronunciation. If possible, use short, simple but complete sentences.
- When using names of people, places or unusual terms, lip-reading is very difficult. If the deaf person is unable to do so, the word can be written in block letters.

Other **inclusive approaches, supplies and supports that can be given by the teacher to help and improve the experience of deaf and blind children** are:

- Make students experience problematic situations also through graphic representation, dramatization, to encourage the association of actions to mathematical symbols. For the deaf and blind students, it is fundamental

to learn by doing associated with mathematical symbols and then visualize it.

- The student should know in advance the topic of the lesson and have access to instructional materials and resources provided through textbooks, audiobooks, the Web, and electronic media.
- A certain flexibility in the time needed to achieve the objectives identified and interventions that allow the transition from concrete to abstract are appropriate, using a wide range of representation models such as concrete experiences, structured material, images, laboratory activities and computer and multimedia technologies.
- In addition, discovery can give satisfaction and therefore motivation to learn and initiate autonomous learning.
- The use of technological tools such as interactive whiteboards and tablets. Among the possible teaching technologies, which are found to be a useful tool for breaking down communication barriers, are multimedia tools.
- Instruction should be given through simple language.
- Use of rubrics and specialized content vocabulary.

Specific for deaf children:

- The teacher can learn to read from the face and posture whether the deaf child is following the explanation or not.
- Computer and subtitles, because they use the visual channel and not the acoustic one.
- Self-correction decreases the deaf student's sense of humiliation due to continuous corrections which, if continued over time, lowers the sense of self-esteem necessary for the construction of autonomy.
- Using concrete and directly visible exercises helps the child to form the first mathematical concepts that will be the basis for later learning. With

the deaf child it is therefore very important to use the visual channel as well, because this channel can clarify ambiguities of the oral code. In addition to being the preferred channel of communication for the deaf, the visual channel is a cognitive resource.

Specific for blind children:

- The way in which numbers are written must also be considered. The blind student will acquire knowledge of graphic signs only when he/she has finished studying the alphabet. In fact, to represent numbers, the first ten letters of the alphabet are used, preceded by a special sign called a "number sign".
- It is necessary that the teacher, together with the word, allows the blind student to manipulate the objects taken into consideration, because it is important that the child has the opportunity to make experiences, not to fall into the risk of empty verbalism.
- According to Del Campo (2000, p. 216-217), it is possible to establish characteristics that can be found in all educational materials used by the blind. These characteristics are Transportability, Adequacy to perceptual characteristics, Simplicity, Cost-effectiveness.
- Very often, even in the normal dynamics of teaching mathematics, there is recourse to particularly useful aids, such as logic blocks, because the child, manipulating, internalizes concepts that would otherwise be difficult to understand.
- With the use of sophisticated computer programs and Software for Learning Mathematics we can support the child in acquiring knowledge

Today, teachers benefit from excellent multimedia materials that enable students to personally interact with the computer, even converse with it.

4.4 Alternative pedagogical approaches

Ensuring that the learning experience for all learners is enjoyable and satisfying is one of the aspects to take into account when developing an approach. The opportunity to experience and appreciate the fun of exploring mathematical problems and the satisfaction of arriving at a solution is a unique situation that should not be ignored. These experiences and activities should arise from children's interests, questions, concerns and everyday experiences.

One of the foundations of alternative pedagogical approaches is to use approaches in the development of emergent literacy and numeracy skills that complement learning in other areas and be child-centred, broadly-based, prioritise play and reinforce the concept of the child as an active learner, the importance of treating all children as if they already have knowledge and experience, taking account of the child's strengths, interests and previous experiences and use these as contexts for new learning. All learners should have the opportunity to engage with learning approaches, including cooperative learning, differentiated learning, active learning, and problem-solving activity, which we know not only contribute to more effective learning, but increase learner's participation and enjoyment of learners in the process.

In addition, it demands that in schools there is an 'ethic of everybody: teachers have both the opportunity and responsibility to work to enhance the learning of all' (Florian & Linklater, 2010, p. 372).

Learning by problems, for example, which is part of inductive teaching, consists of starting from a problem and, through the observation of a certain finite number of facts or events or particular experiences, arriving at a resolution. The method of teaching by problems allows students to learn to solve, gradually, increasingly complex problems that allow them to acquire

cognitive skills at a higher and higher level. The learner is, therefore, at the centre of the process;

With this method you can also develop some fundamental aspects of personality such as:

- Responsibility
- Autonomy
- Self-confidence
- Self-esteem
- Cooperation with others
- Solidarity
- Decision-making skills

Research in the field of Children with Hearing Impairment has demonstrated that 'deaf children have different knowledge, learning styles and problem-solving strategies than hard of hearing children. Teachers need to know how their deaf students think and learn if they are to accommodate their needs and utilise their strengths' (Marschark & Spencer, 2009, p. 210). Recommendations for deaf and hard-of-hearing children include recognising their visual-spatial orientation, which they do not always apply, and their relative lack of confidence in problem-solving. 'It is clear that modifications in curricula and teaching strategies are required if deaf and hard-of-hearing students are to develop to their potential in the important areas of maths. Interventions that have shown promise include those which focus on building problem-solving skills through producing schematic illustrations emphasising visual-spatial over verbal activities (Nunes, 2004).

Access to a mathematics curriculum for children with visual impairment often hinges on specialist teacher knowledge of the unique aspects of mathematics education for such children. This includes the use of calculation with abacus or braillewriter, talking calculator, concrete materials

and tactile displays and teaching of the Nemeth Code (Kapperman et al, 2000). A focus on mathematical language and its accuracy by the educator is also stressed.

The conception of a Universal Design for Learning (UDL) is a step towards the expansion of inclusion in education for all removing barriers created by CAST, an American organization. It is not a unique universal solution to all students' problems, but it is a flexible approach that can be personalized by educators and adapted to the classroom that they have. It helps educator to understand the needs of different students by planning lessons with some flexibility to allow adaptation, instead of the traditional planification that assumes that all pupils learn in the same way, in a "one size fits all" manner. The creation of class profiles helps the lesson planning in order to know the weaknesses and strengths of the group, it is a student-centered approach. To develop the UDL approach it is easier if teachers collaborate and provide feedback in order to share the effects of the UDL approach.

The first UDL principle is representation, which means the provision of multiple ways to represent content instead of relying upon only on one type which is often the case with textbooks, it is important to use video and audio representation as well. The second principle is expression, the way that the student might express themselves is important because traditionally it is most frequently focused on a paper exam but students should also be able to make a presentation for example. The third and last principle concerns the student's engagement in learning and this principle can be used by adding gamification, experience approach, or by relying upon multisensorial activities. The main benefit of using multisensorial activities is the possibility of including blind and deaf children. Multisensorial activities help students with SLD to be more motivated and engaged in learning, children will sustain their attention longer if they are doing something that interests them.

The collaborative learning approach and Inquiry-Based Learning Approach can be complimentary to UDL. The collaborative learning approach is based on the premise of creating profile groups that can work together based on their strengths and likes, it is important to pair children with the same level and pace of development and according to Vygotsky, learning is a social experience (Cooperative Learning and Support Strategies in Kindergarten, n.d.). The educator assumes a very important role as the person responsible for preparing the environment for the children to learn (Ibid, n.d.). The educator is the figure who encourages children to develop their problem-solving abilities and critical thinking. As a result, this method should help develop interpersonal, social, and communicational skills. This approach is inclusive because it takes into account the profile of all pupils and respect their barriers while being part of the same class. While working together children can help one another and compare their knowledge, this way children are active participants in their learning process (Ibid, n.d).

Inquiry-Based Learning Approach is a flexible and dynamic method based on the scientific learning process that follows the scheme: orientation, conceptualization, investigation and conclusion. It promotes a deeper understanding of scientific concepts promoting and at the same time a spirit of self-direction and independence, while pupils learn at their own pace. This method helps children learn how to ask scientific questions while being guided by the educator (Inquiry-Based Science Learning, n.d.). Inquiry-Based Learning Approach allows students to struggle positively since the process is more important than the answer in a scientific context. This method is particularly inclusive for students that cannot learn with the traditional method of memorization-reproduction. Educators might use tools to help visualize the questions generated by students for example idea boards, posters, and scientific notebooks where they can record drawings

and photographs. These tools might motivate students to use scientific concepts through discussions, observation and experiences.

UDL, Collaborative learning approach, and Inquiry-Based approach are not exclusive, educators should mix and match alternative pedagogical approaches to traditional teacher-centered learning to accommodate all types of learners, paying close attention to students with disabilities. Therefore, it is important to have some flexibility while planning classes in order to get to know the students' profiles and adapt the lessons to include all learners.

A student-centered approach is often welcomed by students with SLD because it is engaging and empowering. This approach helps avoid future negative psychological effects on students and the earlier it starts the better because they are not passive learners. This approach allows students to understand their interests and needs because they take part in the planning and implementation of activities with the teachers steering the activities.

Dan Finkel's work is focused on bringing a positive perception of mathematics to ensure children fall in love with it. He therefore, created **five principles to teach mathematics**, of which the fifth and most important principle is play.

Introducing play in mathematics is beneficial to all learners. It helps to avoid future math anxiety and self-confidence difficulties while dealing with problem-solving. The neuroscientist Norman Doidge has written that our brains reorganize every day and it is highly adaptable, being capable of developing different pathways. If children with SLD have an early diagnosis and adapted intervention they can do well in school decreasing overall difficulties (There is a Better Way to Teach Students with Learning Disabilities, n.d.).

4.5 Best Practices to approach simple math concepts

For an equitable curriculum that is inclusive to all children, it is important to consider the needs of children with intellectual, cognitive, and developmental difficulties but also children that are talented in math. It can be quite challenging to have an inclusive curriculum for such a large spectrum of learners varying from children with difficulties and/or mental delays that need constantly to catch up to others to mathematically talented children that need to have their potential met to not become disaffected by mathematics (Mathematics in Early Childhood, n.d.).

Multisensorial strategies that grab pupil's attention, such as using an object to a point, giving emphasis on gesture and using rhythm are beneficial while teaching a child with moderate intellectual or developmental difficulties (Mathematics in Early Childhood, n.d.).

Children with hearing impairment vary from hard of hearing children, the first step is to understand how they learn and to tackle self-confidence while problem-solving. Typically, hard-of-hearing children have difficulties relating bits of information and identifying relationships, also with visual-spatial orientation. These difficulties do not apply to all hard-of-hearing children; therefore, educators must understand the learning style of the child and adapt. The teaching of mathematics to children with visual impairment often relies on specialists and supporting technology such as tactile displays, talking calculators, and others (Ibid, n.d.).

Teaching mathematics for all also means considering cultural and social backgrounds. For example, the child's first language might differ from the one spoken in class and that can therefore have an impact at the beginning of schooling. To counterbalance math language barriers, the educator can expose the children to formal and informal mathematics contexts, emphasize in the teaching of mathematical language and concepts,

plan the use of mathematical concepts in problem-solving situations and also in other areas (Mathematics in Early Childhood, n.d.).

In the Mathematics Instruction for Students with Learning Disabilities or Difficulty Learning Mathematics guide for teachers there are seven **recommendations that can be effective for students with SLD**:

1. Teach students using explicit instruction regularly verbalizing the instructions introducing mathematical reasoning.
2. Teach students using multiple instructional examples, teachers should plan out thoroughly effective instructions with multiple examples.
3. Have students verbalize decisions and solutions to a math problem, this is to encourage students to think-aloud helping to solidify skills and strategy.
4. Teach students to visually represent the information in the math problem, graphic representations mixed with explicit instructions often achieve better results.
5. Teach students to solve problems using multiple/heuristic strategies, this strategy helps organize the problem and gives freedom to the student to choose their strategy.
6. Provide ongoing formative assessment data and feedback to teachers, this strategy can help teachers understand the students' rhythm, difficulties, and disabilities in order to adapt the planning.
7. Provide peer-assisted instruction to students, a collaboration between students is beneficial but cross-age tutoring seems to be a better choice when dealing with students with SLD than within-class tutoring (Jayanthi et al, 2008).

Ladson-Billings, an American pedagogical theorist and teacher educator, lists **six ways in which a curriculum can be inclusive to teach simple math concepts for all children**:

- "The importance of treating all children as if they already have knowledge and experience that can be used as a foundation for teaching.
- The creation of a learning environment that allows children to move from what they do not know to what they do know.
- A focus on high-quality mathematics learning rather than on 'busy' work.
- The provision of challenging tasks to all children.
- The development of in-depth knowledge of children and subject matter.
- The fostering of strong teacher-child relationships" (Mathematics in Early Childhood, n.d.).

The shift of focus in the teaching of mathematics towards the normalization of struggle and freedom to think can be implemented from a young age, this allows children to have challenging tasks that will help them better develop problem-solving skills and have an in-depth knowledge of the process behind a mathematical problem. For this to happen it is important to change the pace and the idea that students should have answers as fast as possible because mathematical thinking is a process, and this struggle can lead to brilliant and creative ideas instead of memorization and repetition.

Chapter 5

The recreaMATHS approach

5.1 Understanding the recreaMATHS approach

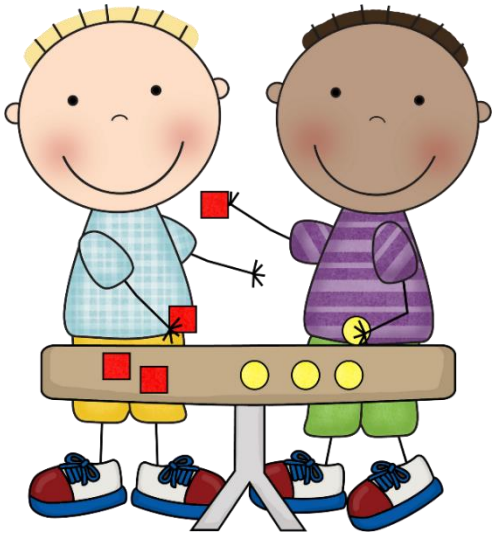
So, what is a 'recreaMATHS approach'? How can we recreate the approach in learning mathematics in kindergarten?

In this chapter, we will give answers to the above questions and try to explain to you in more depth about the recreaMATHS approach. Part of the recreaMATHS approach is based on the wider mathematical competence in kindergarten. By considering how a child learns in general, we can determine a way on how to teach mathematics to kindergarten children. Teachers can identify teaching approaches that avoid overpressure and negative attitudes towards the learning process of a child. Learning is a complicated development, where each individual child learns in a distinct way and evolves this learning at their own pace. There are many aspects involved in this process, however, all children share a developing brain and body, and can be greatly influenced by their experiences – both social and cultural – which can help in the learning process (Gifford, 2005).

Source: https://www.pngkit.com/view/u2e6w7r5i1e6u2r5_math-cartoon-png-number-talks-clip-art/

Learning can be considered as an 'emotionally charged business', which either involves excitement or change to self-esteem. Additionally, children of a young age, do not learn effectively. This can vary on different factors such as, losing their concentration easily, become bored, uncomfortable, or even anxious. Nonetheless, children enjoy practicing new skills and techniques; that is why children like to constantly count things or shout out numerals' names with such emphasis. With this, we can conclude that the concept of practicing is an important skill. Skills can become automatic, setting free some mental space that helps in learning new things. In the social process of learning, 'imitation and instruction play a leading role'. This sentence conveys that we need to provide examples and opportunities where children learn through observation, reflection, instructions, and rehearsal. Young children normally do this automatically and enjoy doing it; spotting similarities or objects repeating themselves (Gifford, 2005).

To understand the recreaMATHS approach, we need to understand what children enjoy doing best. Young children enjoy representing things by illustrating them and this also involves thinking spatially. Referring to Gifford's example: 'if children choose to represent a roof by a triangle, they must have identified that both shapes have sloping



Source: https://www.pngkit.com/view/u2q8r5i1o0e6i1w7_love-math-clipart-free-images-2-clip-art/

sides.' Representation is a significant learning process and in order for children to start representing things, it is essential for them to recognize some key features. Representation can be active or visual, it can either involve words or symbols; it does not affect the process of learning. The recreaMATHS approach encourages engagement. For example, guessing how many teddy bears fit in a box or what 3-dimensional shape can be hidden in a cyclical box, helps engage young children and absorb information about shapes, sizes, while playing. Random guessing – or otherwise in a mathematical term – predicting, focuses the attention of a child and can involve visualization during their thinking process. Finding out the correct answer reflects feedback to the children and is therefore used as advice in the following attempts (Gifford, 2005).

Moreover, an important teaching method is by providing a range of examples, challenging children's' mathematical misconceptions, displaying some confusion, modeling errors. This methodology may encourage children to ask questions such as 'What if…?' and 'How many different…?', which may help them test what their boundaries of ideas are and may prevent possible misconceptions. A part of the inventive procedure is the combinatorial play – taking two unrelated things and putting them together to generate new ideas – or associative play – a form of play in which a group of children participate in similar activities – at any age and level. Thus, children may be encouraged to play with distinct mathematical concepts by providing open-ended frameworks and to support the exploring of alternative potentials and new connections. Gifford suggests that a young child's difficultness with mathematics might be due to the lack of the 'awareness of executive process' such as the process of memorization and the calculation procedure (Gifford, 2005).

"very often young children have an almost deeper perception of what they're seeing", Fresno State professor Kim Morin.

Some significant mathematical teaching approaches for kindergarten students may therefore be summarized as follows:

- Demonstration and instructions
- Connecting and exploring by providing examples which will inspire children to test their ideas
- While discussing children may be encouraged to use mathematical language
- Encouraging representation and visualization
- Posing a problem, encouraging prediction through play, and giving feedback
- Encountering errors and misconceptions
- Demonstrating and encouraging the reflection of thinking

Multisensory Learning:

Moving on to multisensory learning. When learning mathematics, young children use mostly all their senses: for example, researchers have found that young children can detect the changes in numbers with sounds and by visualization. Young children may use their whole-body movements to express or designate things. This can relate to the idea of action 'schemas' – refers to the fundamental concept of intellectual development and to the variability of practices and languages for representing consecutive decision-making problems (Seel, 2012) – or spatial behaviour patterns. Children tend to develop a range of movements, such as up and down, round and round, which they usually repeat and for example can use in their drawings. The use of gestures of visual images appears to be principally effective for counting numbers, that builds on children's capability to identify

the number of things without counting. For example, for children to remember and recognize numbers as optical patterns, objects such as dice and dominoes may help them visualize numbers and symbols.

Source: https://www.pngkit.com/view/u2q8a9i1i1w7q8i1_kids-math-png-maths-fun/

Embedding mathematical discourse and reasoning through a child's ordinary experience and child's-initiated play. Child-centred VS Group experiences in recreaMATHS:

As a recommendation for the learning of mathematics in kindergarten – preschool – both combined and focused establishment have been suggested. Combined provision, is connected with a 'math is everywhere' method – which among other things – includes child-initiated play – a play where children choose what and how to play and who to play with (Drew, 2020) – and adult-led play – activities and a routine planned by the teacher (*How Play Is Structured in Early Years Settings to Promote Development*, n.d.). The planning of mathematics is a crucial and significant step. In this case, the plan and the teaching approaches should contain an equilibrium of open-ended and organized activities along with a combined and focused child-initiated and adult-led play.

Creating a mathematically rich environment relies for support on various examples, which depend on wise ideas based on mathematics and

multisensory resources such as technology. The everyday use of mathematics for various distinct purposes can be depicted by this mathematically rich environment (Gifford, 2005).

It appears that a child needs time to understand and become familiar with the mathematical concepts – this includes numbers, measuring tools, or shapes – before introducing it to them. For example, before understanding or even learning about the value of numbers, children need to practice their counting so that it develops into an automatic process. Furthermore, when children become familiar with a concept – in this case, we will use as an example shapes of blocks – through practice, children could use them to build more complex designs, in both structure and pattern. Hence, a child needs to feel that they have several opportunities and are encouraged by the teachers to familiarise through practice, the mathematical application of problem-solving, such as examining by counting if the colour pencils are shared equally to each child. Even though, by using the phrase 'Mathematics is Everywhere' teachers can create and plan activities in which children do not realize they are using mathematics, it seems to be difficult for the teachers or the children to recognize the mathematics in everyday circumstances.

The majority of kindergarten children find mathematical activities stimulating, pleasant, and relevant. What child does not get excited by hearing large numbers, or enjoy identifying patterns, and matching the shapes? Young children feel satisfaction when they learn how to count or when they begin recognizing numbers. Moreover, children, when still young, find it enjoyable to explore all distinct possibilities and may even create a new method or unique solution to a problem without realizing it. Therefore, mathematics seems to be relevant to young children, all we need to do is encourage them to 'think out of the box'. In order for children to see all the

prospects for mathematics in distinct contexts, teachers could create engaging ideas focused on ways of making math easier to comprehend.

Source: https://www.pinterest.com/pin/539657967853904212/

As mentioned above, activities which are both child-initiated and adult-led are significant in the learning of the mathematical process. Non-competitive and less directive teaching approaches are more effective when teaching young children, since, this allows children to engage more actively and be in control of the teaching strategy. Teachers need to plan a range of distinct interactive methods that consider the greater inequity of influence between kindergarten children and teachers and instead plan activities that protect children's self-esteem. To understand how a child absorbs mathematics partially depends on how the teacher absorbs mathematics and learning in general. The learning process varies between individuals due to several different reasons. There are several critical developments in the learning process of mathematics which scientists do not understand yet. For example, it is not yet known how a child's brain works with the non-verbal images of numbers or in which ways this is combined with counting, or how an abstract idea can be developed by this. Additionally, how are a child's visual images of objects and shapes related to this child's understanding of its properties? No scientist can give an accurate answer, but a teacher, on

the other hand, can offer valuable insights by observing this process of learning and thinking mathematically on an everyday basis.

Establishing a holistic approach; asking, listening and checking:

Source: https://www.pngaaa.com/detail/1490809

Gifford, suggests that teachers should take into account, the importance of children learning holistically – can be defined as the way of providing support to a child as a whole, with both mental and social factors (emotional, physical, social as well as spiritual wellbeing) considered (*What Is a Holistic Approach? - Principles for Effective Support*, n.d.) – which therefore recommends teaching while taking into account all aforementioned factors (Gifford, 2005). One of the most important key features of learning holistically, is to make no assumptions such as how you as an individual, interpret a situation is the same as the person opposite you. For a teacher to be able to provide holistic learning there are three important skills: asking, listening, and checking (*What Is a Holistic Approach? - Principles for Effective Support*, n.d.). This means respecting a child's curiosity in their unique way of learning, helping them figure out their strengths and both empathize and sympathize with their concerns. Kindergarten children have the ability to visualize things, and have the capability to develop this even further. For instance, children can distinguish numbers without counting which implies that children understand that a number could be made-up in distinct ways. In addition, children can instantaneously recognize, or subitize

– a term 'invented' by Piaget, a theorist, and can be defined as a skill to immediately recognize the total number of items in a group without counting (*Subitising and Early Number Sense in Early Years Children - Yellow Door*, n.d.) – numbers, by hearing for example. It is broadly known that music helps the memory and some rhythmical actions may even help children count, implying learning through music or dance. Several different ways of learning may seem promising, nonetheless, they need to be studied in depth. One example is that children represent mathematical concepts, such as numbers, in a unique way. In our days, children are most motivated by technological resources, which include robots, 3D print, e-books, etc. It is unclear for a teacher how these resources could be developed in an educational setting. This is where recreaMATHS steps in.

Approaching mathematics through story-telling and embodied experiences; the activation of the physical body and other means of embodied and oral communication as a means of consolidating knowledge through experience embodied and oral communication. The recreaMATHS mathematical story-telling in kindergarten:

According to Balakrishnan (2008), there is a complicated connection between the imagination and emotions of a child, and when these emotions are involved with the material, then children usually make use of their imagination. Therefore, since stories can be linked to a child's imagination, they have the ability to provoke emotional responses. Additionally, one can argue that narratives can be connected in a complex way with how the mind works. For instance, human beings "think, dream, and perceive the world in terms of stories". Mathematics educators spend a lot of time in developing a child's logical and analytical ability but spend no time in developing one of the most powerful abilities the children have - imagination. A good story stirs the reader's emotions and offers them the opportunity to use their mind in an experimental way, to activate their emotions, encourage them to learn

and enjoy at the same time, and to "add depth to their days". By observing children while playing, we can examine the way children play and adopt characters from the stories they hear and rebuild scenes that capture their imaginations. By examining the world in such a way, children are able to explore as well as relate to several topics and subjects the world might throw at them (Balakrishnan, 2008).

In the past decade, there has been an interest in the role of stories used in mathematical education, both as a cognitive tool for the understanding of mathematics and as a means of knowledge dissemination. Children tend to understand the more difficult math concepts through the context of a story and only after they are engaged in activities that support the idea. We can conclude that stories play an important role in the establishment of meaning (Balakrishnan, 2008).

"Using mathematics to tell stories and using stories to explain mathematics are two sides of the same coin. They join what never should have been separated: the scientist's and the artist's ways of uncovering truths about the world", Robert Frucht.

Similarly, research on the education of mathematics has shown that the child's actions influence how they think and how children think influences their actions. Therefore, embodied cognition – the idea in which the features of 'human cognition' are shaped not only by our brains but also by other parts of our body – plays a significant role in communicating the mathematical concepts and ideas, as well as in allowing children to evolve and experiment their ideas. For example, memories created by movement can prepare learners for future action and can be retrieved and used in solving similar tasks in different situations that no longer engage movement, but instead a mental transformation of the motor processes. Furthermore, gestures have been portrayed to improve learning by helping learners

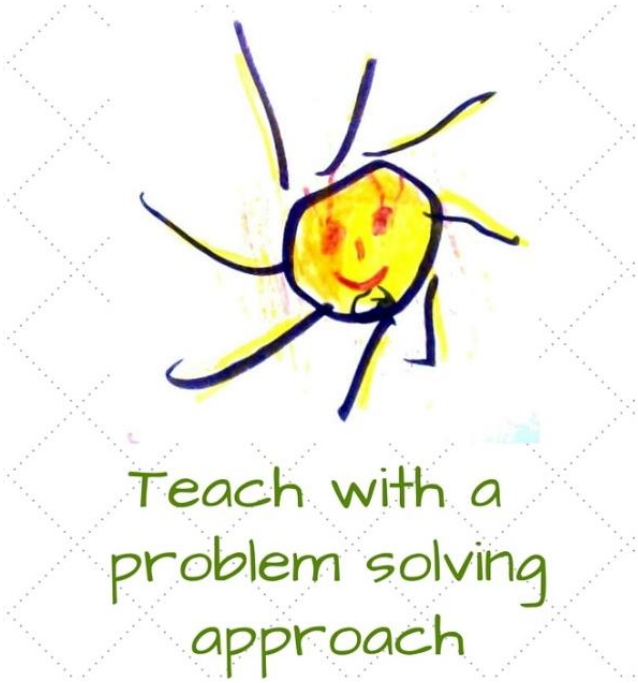
develop existing ideas with less cognitive load. Considering that hands are commonly used to manipulate objects, gestures may offer additional feedback and visual signals by simulating how an object moves if a child is holding it. For instance, gestures may allow the tracking of objects in the mind while mentally rotating it, improving spatial visualization. Moreover, gestures can be also involved in the creation and shaping of new ideas by formulating new ways of thinking through movement. In conclusion, the basic idea of body movement as part of an embodied activity is having a positive impact on cognition by allowing children to learn particular mathematical concepts more advantageously than learning them without movement (Tran et al., 2017).

In addition to gestures and embodied cognition there is sufficient evidence that distinct mathematical aspects are embodied. One of the most recognizable examples is when children use their fingers to count and this helps them further in generating the solution to an arithmetic problem. This is most common in young children, which use their fingers in combination with mathematical tasks. Such a conclusion is not unpredictable given the fact that fingers cover the number range within which children are commonly introduced to when counting. The usage of fingers is a demonstration of embodied cognition. In this example, of children using their finger while counting, embodied cognition has been identified as embodied numerosity. A combination of procedures for deeper levels of processing can be allowed by simply using the senses of sight and sound for a deeper level of processing to create a stronger memory indication that allows children to activate multiple paths in helping them recall that specific memory later on. Memories created through movement can be retrieved and used in solving similar tasks that no longer involve physical movement but instead, a mental transformation of that specific motor developments (Tran et al., 2017).

Movement allows children to bring down their brain's processing power – cognitive load – leaving more resources for the activities – cognitive procedures – which result in the improvement of children's problem-solving abilities. For instance, rather than trying to imagine how an object appears when rotated, the children may reduce this anxiety, concern, of recording information by letting their hands rotate the object and observing what will happen. This can help children in thinking more deeply about the spatial relations and allows them to have a better understanding of math concepts before the transition to a more abstract spatial reasoning. The link between the concrete and abstract physical movements complements children's natural propensity for learning. This is because problem-solving in the real world – required moving through space and manipulating actual objects – emerged before the abstract forms of thought such as mathematics developed. Furthermore, the natural desire to position cognition with real contexts is therefore reflected in the mind-body connections of several mathematical concepts such as embodied numerosity – as mentioned above. By combining the body into the learning experience can, consequently, improve a child's mathematical understanding through the provision of a connection between the abstract notions and the concrete referents. Concluding, the use of fingers is, therefore, a remarkable example in illustrating how a child's physical body features influence the way children process numbers (Tran et al., 2017).

5.2 Problem-Solving Skills as a recreaMATHS approach

Two key types of activity which foster cognitive learning, have been highlighted by research: **problem-solving and play**. These two cognitive learning types can overlap with one another. Problem-solving techniques are used in the form of a game (while playing) or can even be used in the other way round: while playing to use these problem-solving techniques. The concept of problem-solving has been broadly considered as a significant framework while learning. Teachers could provide possible situations that help arise the curiosity and solution-seeking of a child. As a demonstration of the true understanding, detect the spontaneous application of an idea to the new situation. Such an understanding of the problem-solving technique can be achieved through problems integral to activities, such as puzzles, games on the computer, or can even rise out of things children wish to do. The concept of problem-solving, encourage many cognitive processes, such as talking, predicting, and even generating connections to find the solutions.



Source:https://www.kindergarten-lessons.com/kindergarten_problem_solving/

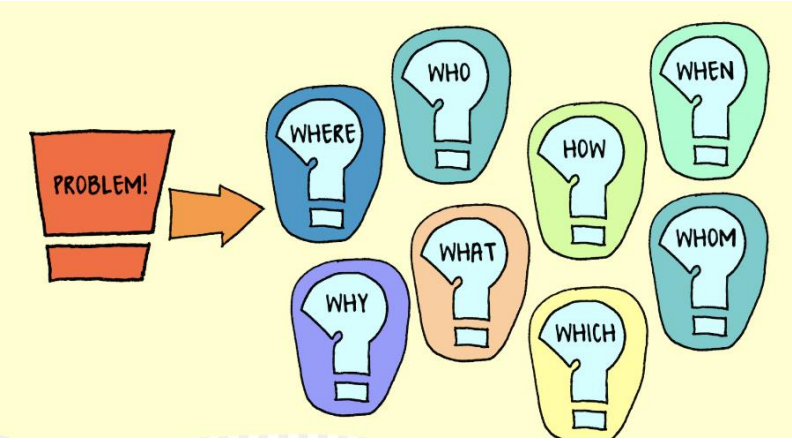
Problem-solving has a key role in learning mathematics. By adopting problem-solving skills, children adopt a specific way of thinking which helps them with almost every math problem they come across. The students that tend to achieve higher scores in math have developed this different way of thinking; this 'problem-solving way' of thinking. Researchers have found that teaching children problem-solving skills – by beginning to teach basic problems – might help improve a child's mental health. Teachers can introduce some basic vocabulary of solving problems in the form of short stories, playing with puppets, or even talk about everyday situations that occur to them. Furthermore, one more approach for problem-solving can be set by asking the children questions. For example, "How would you…?" or "Show me how you could…?". In kindergarten and preschool, these problem-solving activities, may offer to children the opportunity to use previously learned skills to solve current problems while teachers can teach them some new problem-solving strategies. The aforementioned are only a few problem-solving techniques. Through practicing, kindergarten children will be able to identify the problems and challenges, search and find facts that will help them with the problem, think of possible ways to use in solving the problem – such as brainstorming and creative thinking – and lastly use these ideas to test if the problem is solved. The most important step is thinking creatively and having the ability to solve problems.

Source:https://www.liveplan.com/blog/the-best-ways-to-approach-problem-solving-in-business/

Nonetheless, the result, in other words, actually solving the problem, is not the important part. Therefore, reinforce, emphasize and support a child's creative thinking, while talking about all the distinct ways the child tried to solve the emerged problem rather than talking and emphasizing the outcome (Gifford, 2005).

Problem-solving involves using basic approaches in an arranged way to find possible solutions to problems. There are four steps:

1. Define the problem: identify the situation so that the emphasis is on the problem. (helpful problem-solving techniques embrace using charts to help identify the expected steps.
2. Generate alternative solutions: delay the selection of just one solution until there are several clear problem-solving alternatives. Considering several possibilities can significantly improve the value of the solution. For this step, both brainstorming and teamwork are beneficial tools.
3. Evaluate and select an alternative: consider and evaluate before selecting the best alternative.
4. Implement and follow up on the solution ("KINDERGARTEN PROBLEM SOLVING," 2019)



Source:https://medium.com/@iamkanikamodi/steps-to-develop-problem-solving-skills-f9c37dd2cbdc

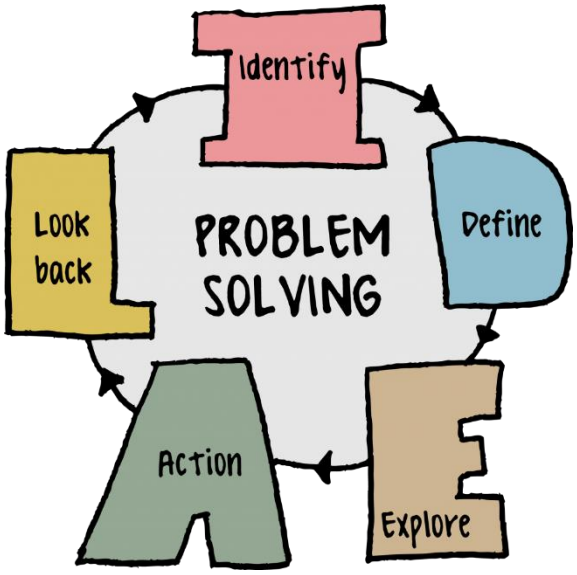
The concept of problem-solving occurs in many different appearances and scopes. Some problems can be rather minor and arise in an incidental manner from activities or on the other hand, problems can be a part of a major project.

So **why is problem-solving important**?

According to Gifford, Piaget (1973) and Vygotsky (1978) have encouraged problem-solving techniques as an important 'vehicle for learning'. They have highlighted the collaborative, as well as guided problem-solving as a significant technique. Children need to learn how to connect the skills they already know to new situations, to overcome current and future difficulties they come across. When problem-solving, children can be encouraged in making new connections with their current knowledge and this may moreover provide motivation for learning. An advantage of the problem-solving technique involves, all the main cognitive learning processes (attention, language, learning, memory, perception, and thought) in visualizing solutions, checking for possible errors, and in all context given instruction along with talking and reflecting. Problem-solving is important since it comprises metacognition – a higher-order thinking which enables one's cognitive process by understanding, analyzing, and controlling it (*Definition of Metacognition*, n.d.) – used in evaluating strategies and solutions (Gifford, 2005).

Problem-solving skills can stimulate a higher level of thinking, including an extended analysis of the problems, a synthesis of important ideas, and the use of creativity when unfamiliar solutions are found. Furthermore, problem-solving usually involves some significant emotional and social learning where successful problem-solving may enhance a child's self-esteem and may help the child grow and develop a 'mastery orientation'. Co-operative problem-solving may help form good relationships

by giving emotional and cognitive support, which have the need for social skills. Some examples of social skills are: gaining entry, both taking and giving advice, and most importantly resolving disagreements. By definition, problem-solving is difficult to pursue and may sometimes threaten a child's – or even an adult's – self-esteem. Thus, a teacher's help and sympathy play a key role in fostering an encouraging and supportive climate for problem-solving in the classroom (Gifford, 2005).



Source:https://transportfutures.institute/key-problem-solving-skills/

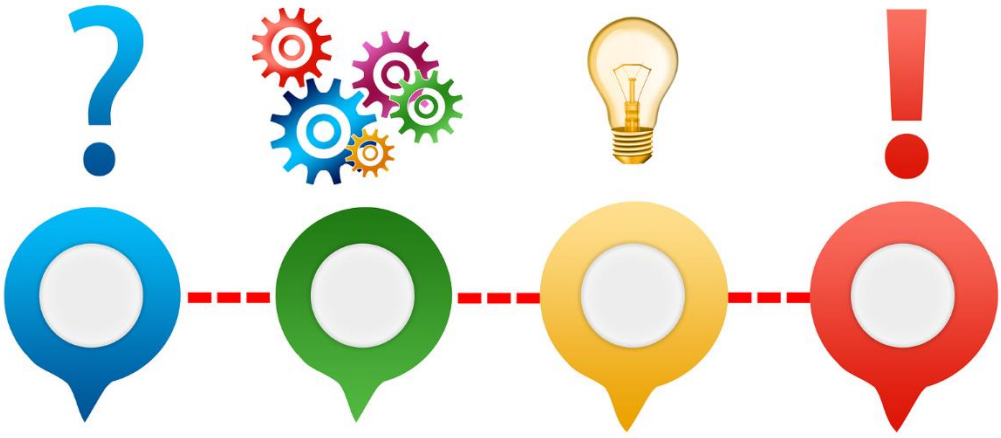
Some successful problem-solving strategies include:

- Comprehending the problem, looking at the problem as a whole, checking if the children have understood the problem by asking them questions and going through it.
- Preparing, planning, and predicting the outcomes of the solution. For example, when playing with blocks, gather them all together before starting to build.
- Monitoring the procedure and progress towards the target. For example, checking that all blocks fit in a box.
- Be efficient by trying all possibilities -without repeating – methodically instead of randomly trying. For example, separate the shapes of a puzzle

into two categories (or more): those the children have already tried and those not tried yet.

- Trying different options of approaches and evaluating distinct strategies. For example, trying the different arrangements of each block/shape.
- Improving and reefing a solution. For example, by solving a puzzle for a second time using fewer moves and being quicker.

All the above strategies involve, among other things, the reflection and awareness of the thinking process. Teachers can advise children to use the above strategies by helping them model these strategies as well as encourage them to talk and compare the different methods they used in the solution. Teachers could also, recommend to children, to 'prepare' a list of alternative methods to select from, conveying a range of expertise. In addition, children need to adopt the confidence of being flexible and familiar with working on the solution of a problem while, using everything they know.



Source: https://ralfw.de/2018/09/two-problem-solving-approaches/

How can teachers help? In general, the notion of 'Problem-solving' is considered to be difficult to teach. However, children can be taught on how to use these strategies, which usually involve mathematical ideas. For example, kindergarten teachers (or adults in general), can teach children to

check their answers using counting and can even provide them with the puzzle- and computer game-based strategies. An example that supports problem-solving is scaffolding (a process through which teachers add support for students to enhance learning and assist them with the difficultness of the task): this covers the provision of support depending on a child's responses, for example, breaking down the problem in smaller steps and helping the children draw attention to the key features. Questioning is considered as a significant strategy to help focus a child's attention, but more exquisite strategies, such as making comments or looking can be also effective. Furthermore, asking the children to describe (talk about), and to demonstrate the steps of what they have done, may help them examine and evaluate the problem. Researchers have found that when a teacher encourages children to check their answers – their steps – means that later on, they would do this themselves. Gifford suggests that it is useful to ask older children some 'self-organizing' questions, of which children could use as examples and base their questions on them. An example of the different stages of a problem could be:

Getting to grips:	What are we trying to do?
Connecting to previous experience:	Have we done anything like this before?
Planning:	What do we need?
Considering alternative methods:	Is there another way?
Monitoring progress:	How does it look so far?

Evaluating solutions:	Does it work? How can we check? Could we make it even better?

Adults who encourage the curious and questioning behavior in children, suggest that demonstrating attitudes may be equally important as the teaching strategies. Children can pursue and become confident in problem-solving, with the help of teachers who recognize the difficulties of problem-solving but can also establish the determination that could help them.

Different types of problem-solving provide a variety of learning experiences for the children, and can also implement an effective teaching strategy that evaluates and clarifies all misconceptions and increases children's understanding.

Prospects for mathematical problem-solving:

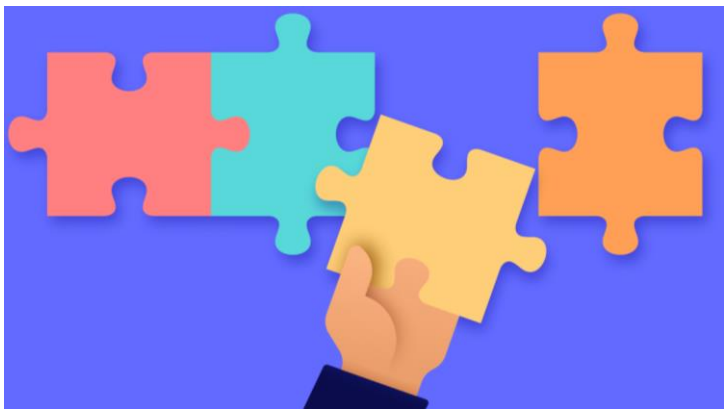
Adult-initiated:

- Preparation – examining that there are sufficient items for everyone, e.g., brushes, puzzles
- Share – examining that all children have an equal amount
- Tidy up – organizing the storage and examining that nothing has been lost
- Gardening – arranging plants and bulbs, and predicting how fast they grow
- Vote – for stories, songs, games and in general vote for the activities
- Plan and set up – a wild garden or new role-play area

- Communication – plans, measurements, invitations with times and maps

Child-initiated:

- Construction plays and model-making materials – finding similar shapes, fitting things together, checking sizes
- Pattern-making materials – generating patterns and setting up rules, such as using shape properties, movements, and positions
- Drawing and picture-making materials – creating shapes to represent things
- Open-ended role-play activities – the 'whatever you want it to be place', for children to construct their own scenarios
- Mathematical utilities for children to use while playing – for example, DIY calendars for making appointments, calculators for working out prices, scales for weighing a baby, etc.



Source:https://www.lucidchart.com/blog/problem-solving-definition

George Polya's book 'How to solve it, (1945)', describes how students can adopt problem-solving skills. Even though his advice is directed to older students (high school and university students), it can be adapted and applied to younger kindergarten students. Polya gives four steps that a student should follow when solving a problem:

1. **Understanding the problem**; this first step is regularly neglected as it is considered too obvious by students. Polya advises teachers to use

some of the following questions to help direct the students in the correct path of understanding the problem. Some questions are:

- What are you asked to find?
- Can you define and describe the problem in your own words?
- Can you think of a diagram or picture that might be able help you comprehend the problem?

If children are unclear as to what needs to be solved, then they will most probably get the wrong answer. Understanding the problem is the most important step and in general, more emphasis should be given to this step.

2. **Devising the plan**; This is a step that is in sync with the policy of encouraging the representation of thinking. This step is the plan, or a translation of the problem in the form of an equation, a diagram, a chart that will help in the process of solving the problem.
3. **Carrying out the plan**; This is the step where you solve the plan of the previous step. It can be considered as an easier step than the previous one since all that needs to be done is to solve the equation of step 2.
4. **Looking back** (review/extend); When problem-solving, it is good to look back and check if you used all the information and review that the answer makes sense. Doing this step can help you predict a distinct strategy that can be used to solve future problems.

By understanding these four steps at a young age, may give students an advantage when solving a problem at a later age (Polya, 1945). To conclude, problem-solving can be considered as the foundation in young children's learning process. Problem-solving must be respected, provided for, encouraged, and sustained in the kindergarten classroom. There are multiple opportunities for problem-solving which may occur in a child's everyday life. In section 5.3 we will elaborate on how, **by using the child's cognitive, social, emotional as well as movement experiences facilitate**

problem-solving and promotes approaches useful in the enduring learning process (Britz, 1993).

5.3 Teach children to view and describe their world mathematically

In recent years, there has been a major advancement in the teaching of 'thinking skills'. The notion of 'thinking skills' has been identified as introducing the 'reasoning, inquiry, and creativeness' of thinking skills. Inquiry combines prediction and creative thinking skills which covers the 'search' for various innovative conclusions. Teachers can use this process by encouraging children to 'think aloud', but further to embolden and stimulate the methods they have used for solving a problem (Gifford, 2005).

"It's more important to demonstrate the logic of math than to memorize rules", Alice P. Wakefield

While school teachers try to teach children a mathematical expression or problem, they usually provide them the correct answer before children make an effort in thinking what the answer might be. Such teachers, eliminate the need for reflection, which can be considered as the one that 'underlies lasting learning'. Teachers should establish three teaching strategies that will help reinforce a student's thinking. These three teaching strategies are:

‘Encourage your students to think’. Although this principle does not sound unreasonable, most teachers usually pass over it. Instead of encouraging each and every student to develop their unique way of thinking – their number sense – every child is expected to solve a mathematical problem in the same way. However, by simply asking the children if they agree with their fellow students’ answers or how they can solve the problem differently, the teacher encourages them to challenge their thinking.

Source: https://worqiq.com/2018/09/open-to-think-pure-thinking-power/

Moving on to **‘encouraging children to think about thinking’**. To support the learning of young children in kindergarten, teachers should encourage their students to think about their thinking. In other words, to help them form a concept of how they ended up with that specific answer. By challenging a kindergarten child to think about their thinking, you help them become more skilled and will therefore be used at doing it. Teachers can start their questions with words such as, “How”; “What”; “Why”; “Which”. One even better idea can be to advise them to work in teams. Children will compare their answers and possibly different methods used, rethink their solutions, and might even defend their logic.

Lastly, **‘encourage representations of thinking’**. Children should know how to represent their thinking with words, pictures, or symbols.

Encouraging the children to reflect and represent their thinking in the classroom will therefore lead to even more thinking and more thinking about thinking.

These three principles can support a more complicated, and at the same time a more practical mental activity. Nonetheless, it seems that an approach that requires thinking is considered a more challenging approach by teachers, rather than learning the textbook by heart. Just as a child that learns to follow specific rules to get to the correct answers does not grow in an understanding of math logic, teachers who follow a specific "recipe" to meet math targets do not grow in the understanding of teaching and learning (Wakefield, 2001)!

Children, teens, and adults, all experience in a unique way, conveying a divergence in the ways we learn best. A teacher's way of handling their students can be impacted by understanding all four different types of learning styles and adapting to an individual's learning. If an individual's learning style has not been activated, this individual might end up lagging behind the rest of his/her classmates. There are four different learning styles: visual learners, auditory learners, reading/writing learners, and kinesthetic learners. The majority of students usually represent more than one learning style each, however, a teacher should adjust the lesson in such a way so that all different learning styles are covered (Malvik, 2020). Furthermore, children have a unique preference in the way they learn: some by hearing, some by doing, others by seeing, some children by reading and others by asking questions. However, all children have one thing in common: all learn best when they can combine objects and topics that find more interesting (*4 Different Learning Styles You Should Know*, n.d.). Planning a lesson that meets all the aforementioned types of learners can be considered easier while children are still in kindergarten.

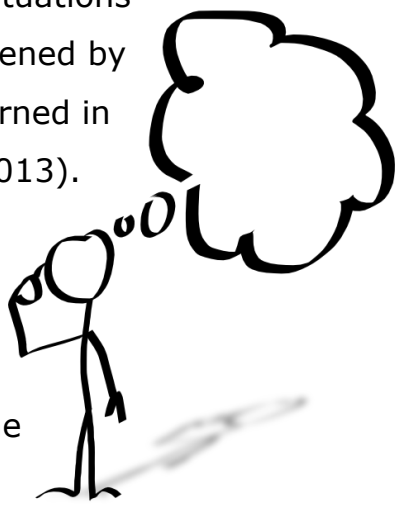
Having all the above in mind, teachers could use this information to teach children how to view and describe their world in a mathematical way. Children could be encouraged to describe in their own words and language, the mathematics they observe and experience in the real world. Research has shown that by applying math concepts to real-world situations during children's play time at school, helps them establish a better understanding of the concepts they are learning. The above recommendations can be implemented using the following steps:

1. Encouraging kindergarten children to represent mathematical concepts, solutions or processes, using informal methods. This step can be completed by connecting mathematical concepts to children's experiences using terms and comparisons that are recognized by the children.

Source:https://www.freepik.com/free-vector/doubtful-boy-white-background_3577417.htm

2. Helping children link their experiences and informal knowledge to the formal math symbols, procedures and vocabulary. Teacher may begin to introduce some formal mathematical concepts, just after children seem to be comfortable with the informal representation of the math concepts. Teachers may proceed by connecting informal and formal representations together.
3. Using open-ended questions in encouraging children to instantly apply their math knowledge. Questions starting with "what", "why", "how", inspire children to think about the informal and formal math vocabulary and concepts.

4. Encouraging children to identify mathematics in everyday situations and talk about it. A child's math knowledge can be strengthened by giving them opportunities to connect the math concepts learned in class to the real world (*Teaching Math to Young Children*, 2013).



Source: https://theheartofthematter-dailyreminders.org/2015/03/02/the-source-of-thoughts/

To conclude, mathematics is a subject that attracts children's interest only when children themselves comprehend that mathematics can be considered as a tool that can be used in solving real-world problems. The goal of the recreaMATHS approach is for teachers to **transfer to the kindergarten children this problem-solving way of thinking**. Once teachers understand that an international low performance of children in mathematics reflects the need for a 'recreation' in the approach used in teaching mathematical concepts, then there will be an excess in the children's mathematical skills. It is advised, that the approach will be distinct from the traditional approach of the teaching and learning of mathematics to attract the interest of as many children as possible (Gifford, 2005). As mentioned above, one of the best ways for teachers to "mathematize" kindergarten children, is to use mathematical concepts in their daily routine. For example, as games or in a different entertaining way to solve a problem. To sum up, attitude matters, therefore, it is suggested that teachers encourage children to believe in success (Pellissier, 2015).

"Math is the language of logic", Dr. Jie-Qi Chen.

References:

4 Different Learning Styles You Should Know: The VARK Model. (n.d.). Retrieved October 29, 2020, from https://educationonline.ku.edu/community/4-different-learning-styles-to-know

Abba, G. (1995). Ruolo del materiale didattico nel processo di apprendimento e integrazione del non vedente in età evolutiva. Tiflologia per l'integrazione, 5 (3), supplem., 34-36.

Anthony, G., & Walshaw, M. (n.d.). *Effective pedagogy in mathematics*. 32.

Asha K., Jitendra K. H., Beck M., "L'uso degli schemi visivi per la risoluzione dei problemi matematici, in Difficoltà di apprendimento", Vol. 8, n. 1, Lehigh University, Bethlehem, 2002.

Balakrishnan, C. (2008). TEACHING SECONDARY SCHOOL MATHEMATICS THROUGH STORYTELLING. 306.

Bavelier D, Tomann A, Hutton C, Mitchell T, Corina D, Liu G, et al. (2000) Visual attention to the periphery is enhanced in congenitally deaf individuals. Journal of Neuroscience 20:1–6.

Best Digital Book in Maths/Sciences 2018 https://innotechtoday.com/digital-book-world-2018/

Britz, J. (1993) Problem Solving in Early Childhood Classrooms. https://files.eric.ed.gov/fulltext/ED355040.pdf

Candiotta Giusla mateeppina, "la matematica per i sordi", relatore Enrico Dolza. – Tesi dattiloscritta. – Torino: Università degli studi, 2015-2016.

Chiara Nanni , Enrica Partesana and Daniela Rocca "Early Identification and Strengthening of Learning Difficulties A Project of Collaboration

between School and Neuropsychiatry Service", Edizione Erickson, DIS Vol. 1, Issue 1, January 2020

Culture and the arts. (2019). In *Wikipedia*. https://en.wikipedia.org/w/index.php?title=Wikipedia:Contents/Culture_and_the_arts&oldid=926544104

D. Fabbretti, E. Tomasuolo, "Scrittura e sordità" Carrocci, Roma, 2006

d'Entremont, Y. (2015). Linking Mathematics, Culture and Community. *Procedia - Social and Behavioral Sciences*, *174*, 2818–2824. https://doi.org/10.1016/j.sbspro.2015.01.973

Davide Antognazza and Silvia Sbaragli "The Influence of the Teacher's Attitude over the Students' Commitment Mathematics Education Case ", Edizione Erickson, DIS Vol. 1, Issue 1, January 2020

Definition of metacognition. (n.d.). Www.Dictionary.Com. Retrieved October 26, 2020, from https://www.dictionary.com/browse/metacognition

Dooley, T., Dunphy, E., Shiel, G., Butler, W. D., Corcoran, D., Farrell, T., NicMhuirí, S., O'Connor, M., & Travers, J. (n.d.). *Mathematics in Early Childhood and Primary Education (3-8 years)*. 164.

Drew, C. (2020). The 5 Key Features of Child Initiated Play (2020). Helpful Professor. https://helpfulprofessor.com/child-initiated-play/

European Math Museums Questionnaire

Fermat Science: https://www.fermat-science.com/

Finkel, D. (2015, June 13). 5 principles of extraordinary math teaching. Math for Love. https://mathforlove.com/2015/06/5-principles-of-extraordinary-math-teaching/

Finkel, D. (2016, May 11). Hate Math? Make It Fun With Picture Books, Board Games and More. Math for Love. https://mathforlove.com/press/hate-math-make-it-fun-with-picture-books-board-games-and-more/

Francesco Sella, Daniela Lucangeli and Marco Zorzi "The Mastering of the Cardinality Principle in a Sample of Italian Preschool Children" Edizione Erickson, DIS Vol. 1, Issue 2, May 2020

Frye, D., Baroody, A., Burchinal, M., Carver, S., Jordan, N., & McDowell, J. (2013). Teaching Math to Young Children. The Institute of Education Sciences (IES), 1–152. https://ies.ed.gov/ncee/wwc/Docs/PracticeGuide/early_math_pg_111313.pdf

Gifford, S. (2005). Teaching Mathematics 3-5: Developing Learning in the Foundation Stage. http://eds.a.ebscohost.com/eds/ebookviewer/ebook/bmxlYmtfXzIzMzkzM19fQU41?sid=f1519a5c-1edb-4669-8a83-c8f4a4142877@sdc-v-sessmgr01&vid=0&format=EB&rid=1

Ginsburg, H. P., Inoue, N. & Seo, K. H. (1999). Young children doing mathematics: Observations of everyday activities. In J. Copely (Ed.), Mathematics in the early years (pp.88–99). Reston,VA: NCTM

Giulia Franconi and Cesare Cornoldi, "A Study on the Change of the Relationship between Mathematics and Metacognition in Different School Levels" Edizione Erickson, DIS Vol. 1, Issue 1, January 2020

How play is structured in early years settings to promote development. (n.d.). 6.

How to teach kindergarten and preschool math. (2019, June 24). *Kindergarten Lessons*. https://www.kindergarten-lessons.com/kindergarten-and-preschool-math/

Instructional Research Group. (2008, January). Mathematics Instruction for Students with Learning Disabilities or Difficulty Learning Mathematics: A Guide for Teachers. Center on Instruction. https://www.researchgate.net/publication/234623037_Mathematics_Instruction_for_Students_with_Learning_Disabilities_or_Difficulty_Learning_Mathematics_A_Guide_for_Teachers

Jayanthi, Madhavi & Gersten, Russell & Baker, Scott. (2008). Mathematics Instruction for Students with Learning Disabilities or Difficulty Learning Mathematics: A Guide for Teachers. Center on Instruction.

Johansson, M. (2015). *Perceptions of mathematics in preschool "-now we have a way of talking about the mathematics that we can work with."*

Jones, C. (2009). Interdisciplinary Approach: Advantages, Disadvantages, and the Future Benefits of Interdisciplinary Studies.

Khasnabis, C., Motsch, K. H., Achu, K., Jubah, K. A., Brodtkorb, S., Chervin, P., Coleridge, P., Davies, M., Deepak, S., Eklindh, K., Goerdt, A., Greer, C., Heinicke-Motsch, K., Hooper, D., Ilagan, V. B., Jessup, N., Khasnabis, C., Mulligan, D., Murray, B., … Lander, T. (2010). Non-formal education. In *Community-Based Rehabilitation: CBR Guidelines*. World Health Organization. https://www.ncbi.nlm.nih.gov/books/NBK310920/

KINDERGARTEN PROBLEM SOLVING. (2019, February 27). Kindergarten Lessons. https://www.kindergarten-lessons.com/kindergarten_problem_solving/

L' Europe mathématique/Mathematical Europe: Histoires, mythes, identités publié par Catherine Goldstein, Jeremy Gray, Jim Ritter

Le Point Hors-Série, La Méthode Singapour, December 2017

Les Défis de l'enseignement des mathématiques dans l'éducation de base, Michèle Artigue, Unesco report, 2011

Making the Connection between Culture and Mathematics Northwestern University | School of Education & Social Policy. (n.d.). Retrieved December 18, 2020, from https://www.sesp.northwestern.edu/news-center/inquiry/2009-spring/making-the-connection.html

Malvik, C. (2020). 4 Types of Learning Styles: How to Accommodate a Diverse Group of Students. https://www.rasmussen.edu/degrees/education/blog/types-of-learning-styles/

Maria Grazia Palermo, "Matematica e scienze con l'alunno sordo : teaching mathematics and science of to the deaf students"; relatore Enrico Dolza. - Tesi dattiloscritta. - Torino : Università degli studi, 2015 (TESI 136).

Mathematics in Early Childhood and Primary Education (3-8 years), NCCA,

Mathematikum museum: https://www.mathematikum.de/

Mmaca museum: https://mmaca.cat/

Mok, O. N. A. (2011). Non-formal learning: Clarification of the concept and its application in music learning. Australian Journal of Music Education, 1, 11–15.

Navet museum: https://www.navet.com/

Neculae Dinuță - Methodical aspects regarding the interdisciplinary approach of mathematical contents in preschool education. Procedia - Social and Behavioral Sciences 180 (2015) 780 – 787

NMC HorizonReport: 2017 Higher Education Edition

Non-formal education. (1999). https://assembly.coe.int/nw/xml/XRef/X2H-Xref-ViewHTML.asp?FileID=8807&lang=EN

Non-formal education. (2020, June 22). http://uis.unesco.org/en/glossary-term/non-formal-education

Observation Cooperation Information Museums: https://ocim.fr/2020/12/approches-sociales-et-inclusives-des-musees-et-des-lieux-de-culture-scientifique/

Oldridge, M. (2019). The Playful Approach to Math. Edutopia. https://www.edutopia.org/article/playful-approach-math

Papadakis, S., Kalogiannakis, M., & Zaranis, N. (2016). Improving Mathematics Teaching in Kindergarten with Realistic Mathematical Education. Early Childhood Education Journal, 45(3), 369–378.

Pellissier, H. (2015). Why early math is just as important as early reading. Parenting. https://www.greatschools.org/gk/articles/early-math-equals-future-success/

Pigliacampo R.,” Lingua e Linguaggio nel sordo, analisi e problemi di una lingua visivo-manuale”, Armando Editore, Roma, 1998.

Pourquoi l’école a-t-elle enseigné le comptage-numérotage pendant près de 30 années ?, Rémi Brissiaud, september 2014

Press article on maths museums: https://lepetitjournal.com/francfort/a-voir-a-faire/mathematikum-le-premier-musee-interactif-des-mathematiques-giessen-attire-les-visiteurs-77890

Rosy Luca, “Insegnare matematica ad allievi con disabilità acustica = teaching mathematics for deaf student”, rel. Cecilia Maria Marchisio. - Tesi dattiloscritta. - Torino : Università degli Studi, PAS Piemonte - a.a. 2013-2014 (TESI 119).

Samarapungavan, Patrick, Mantzicopoulos, A. H. P. (2011). What Kindergarten Students Learn in Inquiry-Based Science Classrooms. Taylor & Francis Group, 416–470. https://doi.org/10.1080/07370008.2011.608027

Seel, N. M. (2012). Action Schemas. In N. M. Seel (Ed.), Encyclopedia of the Sciences of Learning (pp. 73–75). Springer US. https://doi.org/10.1007/978-1-4419-1428-6_356

Sensorineural Hearing Loss Occurs in the Cochlea | MED-EL, https://www.youtube.com/watch?v=EfzvT_WR0x4.

Simonetta Maragna, “ Una scuola oltre le parole : educare il bambino sordo alla lingua parlata e scritta : con esempi di unità didattica” ; presentazione di Virginia Volterra. - Milano : F. Angeli, 2003 (Cap. 4: Le unità didattiche. Area Logico-matematica).

Soares, N., Evans, T., & Patel, D. R. (2018). Specific learning disability in mathematics: a comprehensive review. Translational Pediatrics, 7(1), 48–62. https://doi.org/10.21037/tp.2017.08.03

Spiteri, M. (2016, August 18). *The benefits of non-formal learning* [Text]. EPALE - European Commission. https://epale.ec.europa.eu/en/blog/benefits-non-formal-learning

St Patrick’s College, Charles Sturt University, & Educational Research Center. (2014a). Mathematics in Early Childhood and Primary Education (3–8 years) Definitions, Theories, Development and Progression (No. 17). National Council for Curriculum and Assessment.

St Patrick's College, Charles Sturt University, & Educational Research Center. (2014b). Mathematics in Early Childhood and Primary Education (3–8 years) Teaching and Learning (No. 18). National Council for Curriculum and Assessment.

Subitising and Early Number Sense in Early Years Children—Yellow Door. (n.d.). Retrieved October 27, 2020, from https://www.yellow-door.net/blog/what-is-subitising/

Teaching Math to Young Children. (2013). 165.

Teresa Ferraiuolo and Rosanna Coseglia, "Playing with Numbers and Letters An Enhancement Experience in an Inclusive Context" Edizione Erickson, DIS Vol. 1, Issue 1, January 2020

Terezinha Nunes, "Teaching mathematics to deaf children" - London and Philadelphia: Whurr, 2004.

Tiziana Raso, "Metodologie didattiche per l'insegnamento della matematica ai sordi : didactic methods for teaching mathematics to deaf students"; relatore Enrico Dolza. - Tesi dattiloscritta. - Torino : Università degli studi, 2015. (TESI 138).

Tran, C., Smith, B., & Buschkuehl, M. (2017). Support of mathematical thinking through embodied cognition: Nondigital and digital approaches. Cognitive Research: Principles and Implications, 2(1), 16. https://doi.org/10.1186/s41235-017-0053-8

Tudge, J., & Doucet, F. (2004). Early mathematical experiences: Observing young black and white children's everyday activities. Early Childhood Research Quarterly, 19(1), 21–39.

UDL: The UDL Guidelines. (2021, April 9). The UDL Guidelines. https://udlguidelines.cast.org.

Una app dedicata ai bambini ciechi per imparare la matematica : https://www.superabile.it/cs/superabile/istruzione/una-app-dedicata-ai-bambini-ciechi-per-imparare-la-matematica.html.

Understanding teaching professionals' digital competence: What do PIAAC and TALIS reveal about technology-related skills, attitudes, and knowledge?, R. Hamalainen et al., 2020

Valentina Franceschina "Il bambino non vedente e la matematica: quali strumenti è opportuno utilizzare per facilitare questo apprendimento?" tesi di laurea in scienze della educazione.

Valentina Mengoni and Laura Maria Castagna, "Comparison between Reading Performances of Children with Specific Learning Disabilities and Learning Difficulties by Using Times New Roman and EasyReading Writing Font" Edizione Erickson, DIS Vol. 1, Issue 2, May 2020

Vodopivec, J. (in press). COOPERATIVE LEARNING AND SUPPORT STRATEGIES IN THE KINDERGARTEN. Metodički Obzori.

Vygotsky L. S., "Pensiero e linguaggio", a cura di Costa A., Giunti, Firenze, 1966.Ø M. C. Caselli, S. Maragna, V. Volterra, "Linguaggio e sordità", il Mulino, 2006

Wakefield, A. P. (2001). It's more importantto demonstratethe logic of math than to memorize rules. 5.

What are strategies for teaching a student with a math-related learning disability? | DO-IT. (n.d.). What Are Strategies for Teaching a Student with a Math-Related Learning Disability? Retrieved January 12, 2021, from https://www.washington.edu/doit/what-are-strategies-teaching-student-math-related-learning-disability

What are typical challenges students with math-related learning disabilities face? | DO-IT. (n.d.). What Are Typical Challenges Students with Math-Related Learning Disabilities Face? Retrieved January 14, 2021, from https://www.washington.edu/doit/what-are-typical-challenges-students-math-related-learning-disabilities-face?323=

What is a holistic approach? - Principles for effective support. (n.d.). Retrieved October 27, 2020, from https://www.health.nsw.gov.au/mentalhealth/psychosocial/principles/Pages/holistic.aspx

What Is Culture? Definition, Meaning and Examples | *Live Science*. (n.d.). Retrieved November 13, 2020, from https://www.livescience.com/21478-what-is-culture-definition-of-culture.html

What is non formal education and why it is important. (2018, September 18). *Dothegap / Sube Tu Experiencia. Descubre Otras Afines. Conecta.* https://dothegap.com/blog/en/what-is-non-formal-education-and-why-it-is-important/

What is non-formal education? (2015). https://infed.org/mobi/what-is-non-formal-education/

What's Dyscalculia? (n.d.). What's Dyscalculia? Retrieved January 15, 2021, from https://www.dyscalculia.org/dyscalculia/what-s-dyscalculia

Yeo, D. (2003). Dyslexia, Dyspraxia and Mathematics (1st ed.). Wiley.

Zisopoulou, E. (2019). Collaborative learning in kindergarten: Challenge or reality? Erken Çocukluk Çalışmaları Dergisi, 3(2), 335–351. https://doi.org/10.24130/eccd-jecs.1967201932113

Erasmus+

The recreaMATHS project has been funded with the support from the European Commission. This publication reflects the views only of the author, and the Commission cannot be held responsible for any use which may be made of the information contained therein.

Project number: 2020-1-FR01-KA201-079971